Wannabe Dipoles and Multigluon Production at High Energy

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Why?

A. Multiparticle correlations at RHIC: Mach cone or "deflected jet" or what?

Kharzeev-Levin-McLerran, Nucl.Phys.A748:627 (hep-ph/040327160) - "correlations in the initial state"

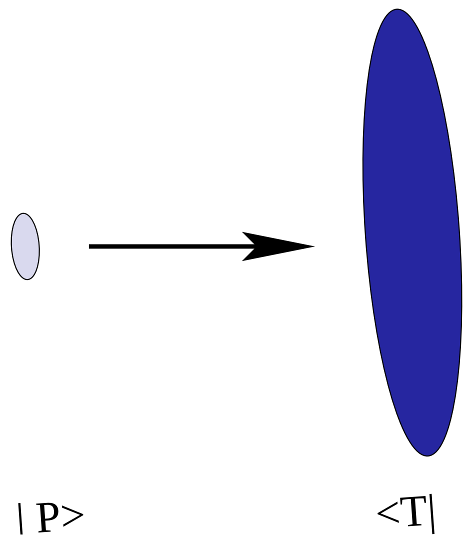
B. At high energy for large P_t much better under control than the total cross section

- WE DON'T HAVE TO CONSTANTLY APOLOGIZE

Jalilian-Marian, Kovchegov: Phys.Rev.D70:114017,2004 (hep-ph/0405266) - double gluon inclusive

High energy evolution - "the JIMWLK equation"

Small perturbative Projectile $|P\rangle$ scatters on a large dense Target $\langle T|$.



$|P\rangle$ has some distribution of color charges - color charge density ρ^a.

$|T\rangle$ is an ensemble of strong color fields α^a.

Energy is high - scattering is eikonal

THE S - MATRIX

The eikonal S - matrix:

$$\hat{S} = \exp\left\{ i \int d^2x \hat{\rho}^a(x) \hat{\alpha}^a(x) \right\}$$

The forward scattering amplitude:

$$\mathcal{S}(Y) = \langle T | \langle P | \hat{S} | P \rangle | T \rangle = \int D\alpha^a \; W^T[\alpha(x)] \; \Sigma_Y^{PP}[\alpha(x)] \, .$$

with

$$\Sigma_Y^{PP}[\alpha] \equiv \langle P | \, \hat{S} \, | P \rangle = \int d\rho \; W_Y^P[\rho] \;\; \exp\left\{ i \int d^2x \, \rho^a(x) \; \alpha^a(x) \right\}$$

and $W^T[\alpha]$ - probability distribution of the color fields in the target.

Σ^{PP} is a function of the single gluon scattering matrix $\Sigma^{PP}[S]$

$$S^{ab}(x) = \langle 0 | a_i^a(x) \, \hat{S} \, a_i^{\dagger b}(x) | 0 \rangle = \mathcal{P} \, \exp\left\{ i \int dx^- \, T^a \, \alpha_t^a(x, x^-) \right\}^{ab} \, .$$

EVOLUTION OF THE PROJECTILE WAVE FUNCTION

We boost the projectile: the boost opens additional longitudinal phase space which is populated by "soft" gluons

$$|P\rangle_{Y+\Delta Y} = C_{\Delta Y}|P\rangle_Y$$

$$C_{\Delta Y} = \exp\left\{ i \int d^2x\, b_i^a(x) \int_{\Lambda}^{e^{\Delta y}\Lambda} \frac{dk^+}{\pi^{1/2}|k^+|^{1/2}} \left[a_i^a(k^+, x) + a_i^{\dagger a}(k^+, x))\right] \right\}$$

The "classical" WW field

$$b_i^a(z) = \frac{g}{2\pi} \int d^2x \frac{(z-x)_i}{(z-x)^2} \rho^a(x)$$

The coherent operator C dresses the valence wave function by the cloud of the soft Weizsacker-Williams gluons:

$$C^\dagger A_i^a(k^+, z)\, C = A_i^a(k^+, z) + \frac{i}{k^+} b_i^a(z)$$

EVOLUTION OF THE S - MATRIX

$$\Sigma^{PP}_{Y+\Delta Y}[\alpha] \;\equiv\; \langle P|C^{\dagger}_{\Delta Y}\,\hat{S}\,C_{\Delta Y}|P\rangle$$

$$= \langle P|C^{\dagger}_{\Delta Y}(a, a^{\dagger}, \hat{\rho})\ \exp\left\{ i \int d^2x \hat{\rho}^a(x)\hat{\alpha}^a(x) \right\}\ C_{\Delta Y}(a, a^{\dagger}, \hat{\rho})|P\rangle$$

It is possible to explicitly:

A. Average C and $C^{\dagger}$ as a function of a and $a^{\dagger}$ over the soft gluon vacuum.

B. Express the factors of $\rho^a(x)$ as functional derivatives with respect to $S(x)$.

ALL SAID AND DONE:

$$\frac{d}{d\,Y}\,\Sigma_Y^{PP}[S]\;=\;-H^{JIMWLK}\left[S,\frac{\delta}{\delta\,S}\right]\;\;\Sigma_Y^{PP}[S]\,.$$

with the JIMWLK Hamiltonian

$$H^{JIMWLK}\;=\;\int_z\;Q_i^a(z)\;Q_i^a(z)$$

The gluon production amplitude

$$Q_i^a(z)\;=\;g\;\int_x\,\frac{(x\,-\,z)_i}{(x\,-\,z)^2}\;\left[J_L^a(x)\;-\;S^{ab}(z)\;J_R^b(x)\right]$$

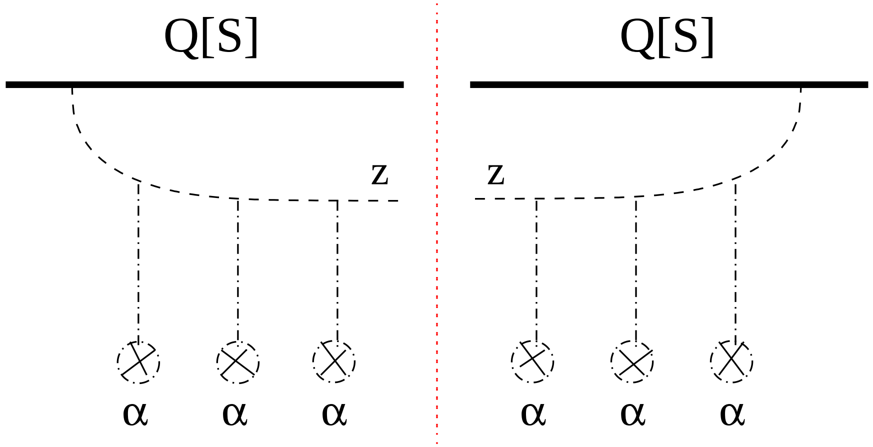
The generators of the left/right color rotations

$$J_R^a(x)\;=\;-\,\mathrm{tr}\,\left\{S(x)\;T^a\,\frac{\delta}{\delta S^\dagger(x)}\right\}\;,\qquad\qquad J_L^a(x)\;=\;-\,\mathrm{tr}\,\left\{T^a\;S(x)\;\frac{\delta}{\delta S^\dagger(x)}\right\}$$

FOR THE FORWARD AMPLITUDE

$$\frac{d}{dY}\mathcal{S} = -\int D\alpha^a\ W^T[\alpha]\ \ H^{JIMWLK}\left[S, \frac{\delta}{\delta\,S}\right]\ \ \Sigma_Y^{PP}[S]\,.$$

Why gluon production amplitude? Before boost the scattering is eikonal - number of gluons is preserved by the scattering. After boost - final state after scattering can contain an extra gluon. So the change in the scattering probability is the probability of producing an extra gluon.



Complementary ways of thinking:

A. The wave function evolves - has an extra gluon at higher rapidity - all gluons scatter eikonally.

B. The S - matrix operator is evolved - it is the same state that scatters, but apart from eikonal scattering there is a probability to radiate a gluon.

THE DIPOLE LIMIT

Suppose the projectile is made of "color dipoles".

The dipole S - matrix

$$s(x,y) = \frac{1}{N_c} Tr[S_F(x) S_F^\dagger(y)]$$

Suppose $\Sigma^{PP} = \Sigma[s]$

The color strucure simplifies immensely

Simple algebra+large N_c limit the JIMWLK equation becomes

$$\frac{d\Sigma}{dY} = -\bar{\alpha}_s \int d^2z \frac{(x-y)^2}{(x-z)^2\,(y-z)^2} \; [s(x,y) - s(x,z)\,s(y,z)] \frac{\delta}{\delta s(x,y)} \Sigma[s]$$

First order equation - the solution is $\boxed{\Sigma_Y[s] = \Sigma_0[s(Y)]}$

with $s(Y)$ solving the Kovchegov equation

$$\frac{ds(x,y)}{dY} = -\bar{\alpha}_s \int d^2z \frac{(x-y)^2}{(x-z)^2\,(y-z)^2} \; [s(x,y) - s(x,z)\,s(y,z)]$$

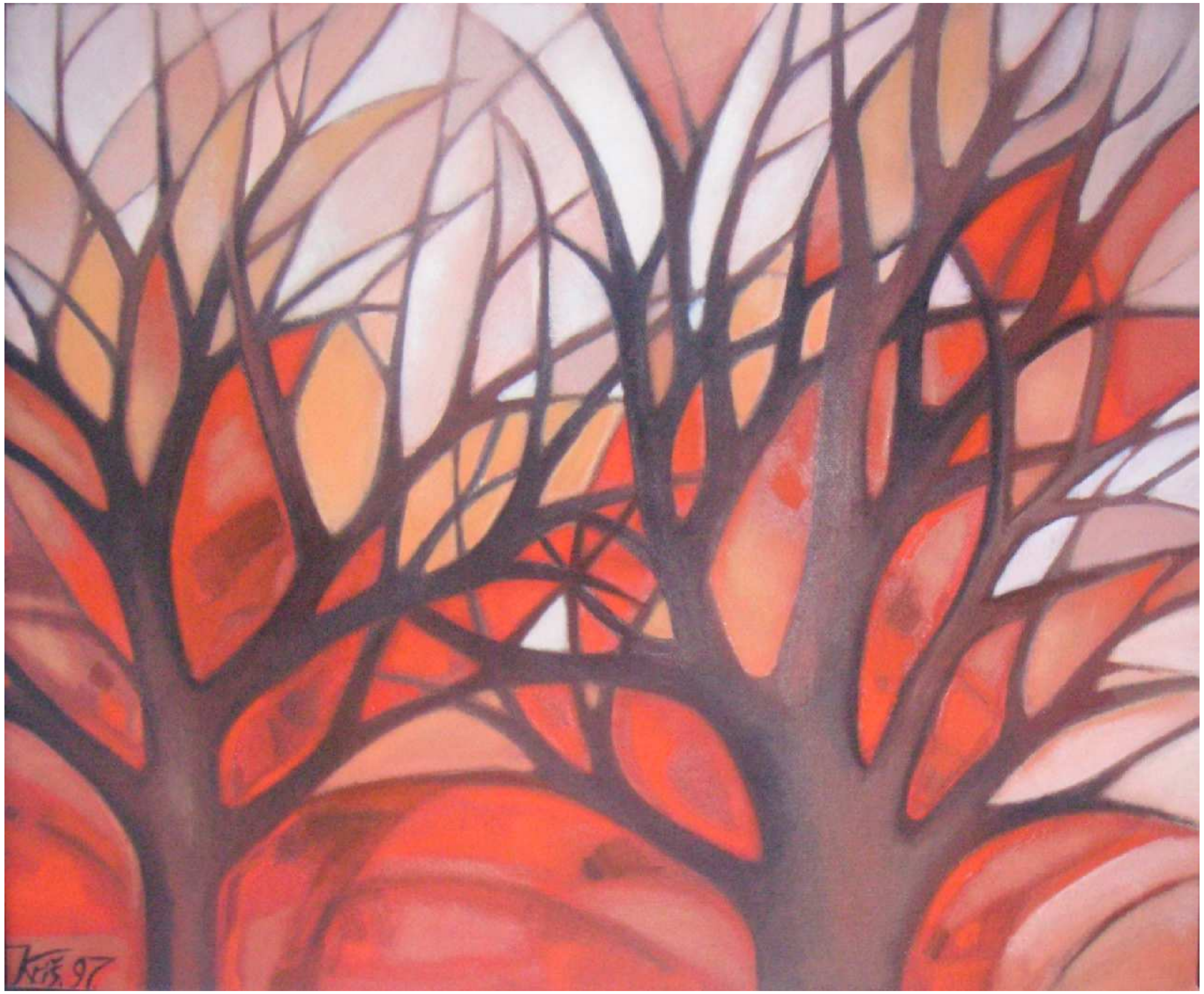


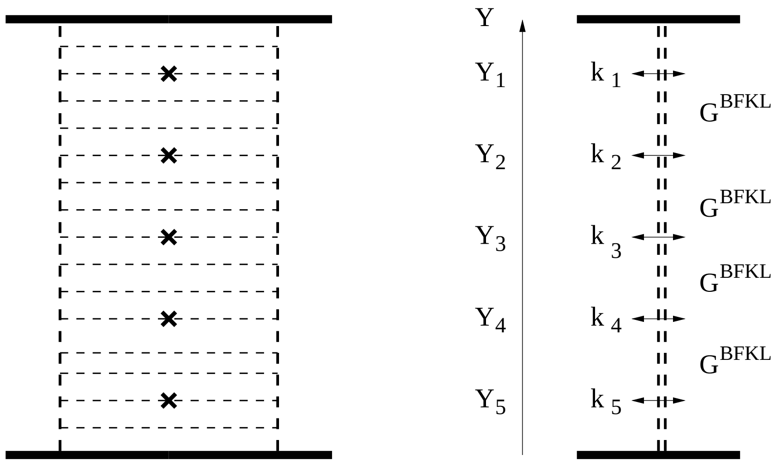
Figure 1: Red Pomeron Trees by Natalija Krisciuniene.

BEYOND THE FORWARD AMPLITUDE

We know that the "perturbative saturation" violates Froissart bound - total cross section is not the best observable.

Hard gluon production should be much better.

For the BFKL approximation:



Very schematically:

$$\frac{d\sigma}{dY_1\,dk_1^2\;\ldots\;dY_n\,dk_n^2} \sim \Phi^T \;\; G^{BFKL}_{Y_n-Y_0}\,L(k_n)\;\ldots\;G^{BFKL}_{Y_1-Y_2}\,L(k_1)\,G^{BFKL}_{Y-Y_1}\;\;\Phi^P$$

GENERALITIES: SEMI INCLUSIVE OBSERVABLES

The total wave function of the projectile-target system before the collision

$$|\Psi_{in}\rangle \; = \; C_Y \, |P\rangle \, |T\rangle \; .$$

THE PROJECTILE HAS BEEN EVOLVED BY THE TOTAL RAPIDITY OF THE PROCESS

The system emerges from the collision region with the wave function

$$|\Psi_{out}\rangle \; = \; \hat{S} \, C_Y |P\rangle \, |T\rangle \; .$$

Calculating any observable O (for example number of gluons) in the final state:

$$\langle \hat{O} \rangle \; \approx \; \langle \Psi_{out} | \hat{O} | \Psi_{out} \rangle \; \approx \; \langle T | \langle P | \, C_Y^\dagger \, \hat{S}^\dagger \, \hat{O} \, \hat{S} \, C_Y \, |P\rangle \, |T\rangle \; .$$

Except not quite: the system has to evolve to asymptotic time $t \longrightarrow \infty$ (account for final state emissions)

$$\langle \hat{O} \rangle \; = \; \langle T | \, \langle \, P | \, C_Y^\dagger \; (1 \; - \; \hat{S}^\dagger) \, C_Y \;\; \hat{O} \;\; C_Y^\dagger \, (1 \; - \; \hat{S}) \, C_Y \, |P\rangle \, |T\rangle$$

Technically it is convenient to distinguish between the target fields in the amplitude and the conjugate amplitude:

$$\mathcal{O}_Y[S,\bar{S}] \;=\; \langle\, P|\, C^\dagger_Y\,(1\;-\;\hat{S}^\dagger)\,C_Y\;\;\hat{\mathcal{O}}\;\;C^\dagger_Y\;(1\;-\;\hat{\bar{S}})\,C_Y\,|P\,\rangle$$

The observable eventually is calculated as

$$\langle\hat{\mathcal{O}}\rangle_Y \;=\; \langle T|\;\mathcal{O}_Y[S,\bar{S}]|_{\bar{S}\,=\,S}\;|T\rangle \;=\; \int DS\,D\bar{S}\,W^T[S]\,\delta(S\;-\;\bar{S})\;\mathcal{O}_Y[S,\bar{S}]$$

Just like for the forward scattering amplitude we can algebraically deduce the high energy evolution

$$\frac{d\mathcal{O}_y[S,\bar{S}]}{dy} \;=\; \lim_{\Delta y\to 0}\frac{\mathcal{O}_{y+\Delta y}[S,\bar{S}]\;-\;\mathcal{O}_y[S,\bar{S}]}{\Delta y}$$

SEMIINCLUSIVE EVOLUTION

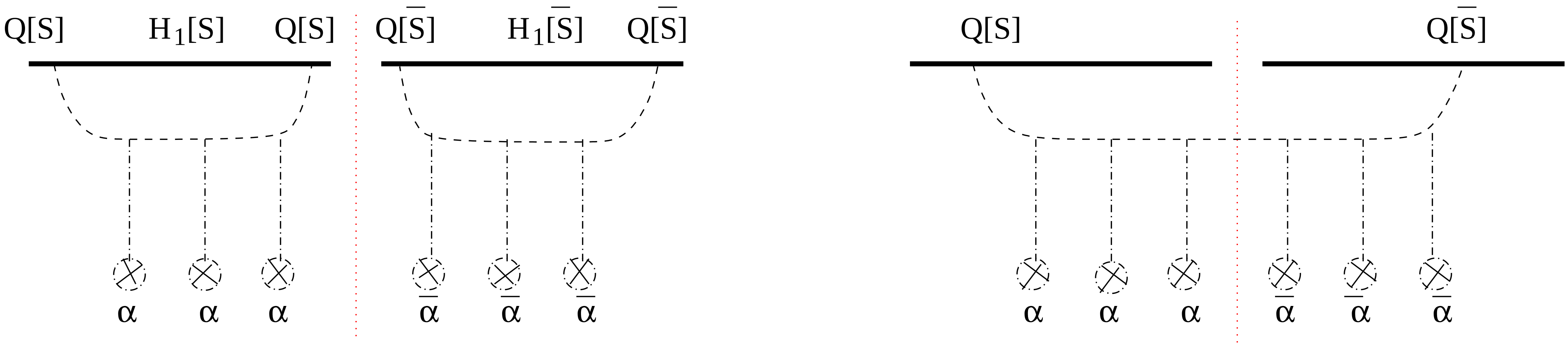
$$\frac{d}{dy}\,\mathcal{O}[S,\bar S] \;=\; -\,H_3[S,\bar S]\;\mathcal{O}[S,\bar S]$$

The Hamiltonian H_3

$$H_3[S,\bar S]\;\equiv\;\int_z \left[Q^a_i(z,[S]) + \,Q^a_i(z,[\bar S])\right]\,\left[Q^a_i(z,[S]) + \,Q^a_i(z,[\bar S])\right]$$

For any observable the evolution operator between rapidity y_1 and y_2 is

$$U_3(y_1,\;y_2)\;=\;Exp\,[-\,H_3\,(y_2\;-\;y_1)]$$



FOR A STRING OF OBSERVABLES AT DIFFERENT RAPIDITIES:

$$\langle \hat{\mathcal{O}}_n \ldots \hat{\mathcal{O}}_1 \rangle \;=\; \int DS\, D\bar{S}\, W^T[S]\, \delta(S \;-\; \bar{S})\, U_3(0,\; Y_n)\, \mathcal{O}_n[S, \bar{S}]\, U_3(Y_n,\; Y_{n-1}) \times$$

$$\times\;\; \mathcal{O}_{n-1}[S, \bar{S}] \;\ldots U_3(Y_1, Y)\, \Sigma_0^{PP}[S^\dagger\, \bar{S}]$$

NOTE:

$\Sigma_0^{PP}[S^\dagger\, \bar{S}]$ - is the "forward scattering amplitude" at initital rapidity in the "target field" $\alpha + \bar{\alpha}$.

In the end though: $S = \bar{S}$, and $\Sigma^{PP} = 1$.

But the presence of $\mathcal{O}_i$ does not allow setting $\Sigma = 1$ right away.

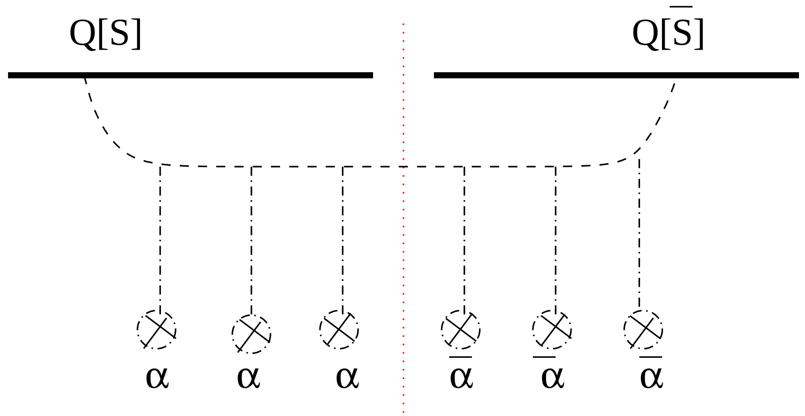
SPECIFICS: GLUON PRODUCTION

The observable

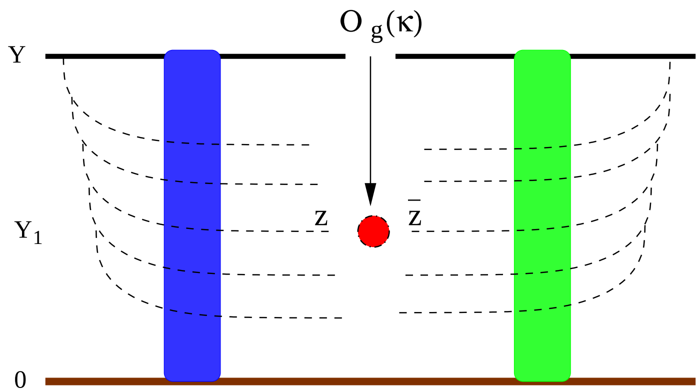
$$\hat{\mathcal{O}}_g \;=\; \hat{n}(k_\perp, y) \;=\; a_i^{\dagger\, a}(k_\perp, y)\, a_i^a(k_\perp, y)$$

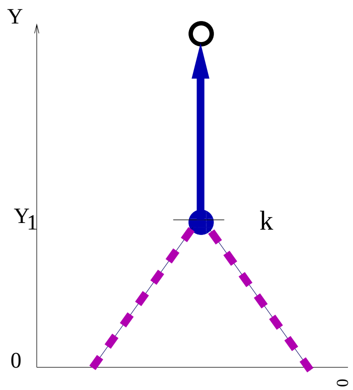
We go through the motions and derive THE OPERATOR

$$\mathcal{O}_g[S, \bar{S}] \;=\; \int \frac{d^2 z}{2\,\pi}\,\frac{d^2 \bar{z}}{2\,\pi}\; e^{i\,k_\perp\,(z\,-\,\bar{z})}\; Q_i^a(z, [S])\, Q_i^a(\bar{z}, [\bar{S}])$$



SINGLE INCLUSIVE GLUON PRODUCTION





Yu. Kovchegov

and

K. Tuchin, 2001

$$\frac{d\sigma}{dY_1\,dk^2} = \int DS\,D\bar{S}\,W^T[S]\,\delta(\bar{S} - S)\,U_{Y_1}\,\mathcal{O}_g^k[S,\bar{S}]\,U_{Y-Y_1}\,\Sigma_Y^P[S^\dagger\bar{S}]$$

$$\frac{d\sigma}{dY_1\,dk^2} = \frac{\alpha_s}{\pi}\int_{z,\bar{z}} e^{i\,k(z-\bar{z})}\int_{x,y}\frac{(z-x)_i}{(z-x)^2}\,\frac{(\bar{z}-y)_i}{(\bar{z}-y)^2}\,G^{BFKL}(x,y;Y-Y_1)\,\times$$

$$\times\,[\langle s_{z,y}\rangle_{Y_1} + \langle s_{x,\bar{z}}\rangle_{Y_1} - \langle s_{z,\bar{z}}\rangle_{Y_1} - \langle s_{x,y}\rangle_{Y_1}]$$

$\langle s \rangle$ denoting S-matrix of a dipole:

$$\langle s_{x,y} \rangle_{Y_1} \;\equiv\; \int DS\, W^T_{Y_1}[S]\, \mathrm{tr}[S^\dagger_x\, S_y]$$

can be deduced from solutions of the BK-JIMWLK eqn

Beyond single gluon is difficult without approximations...

BACK TO THE THE DIPOLE MODEL

Like before - assume that our projectile contains only dipoles, and take $N_c \to \infty$ limit.

OUR ZOO:

The dipole

$$s_{x,y} = \frac{1}{N} tr[S_F(x)\, S_F^\dagger(y)]\,;$$

The "mirror" dipole

$$\bar{s}_{x,y} = \frac{1}{N} tr[\bar{S}_F(x)\, \bar{S}_F^\dagger(y)]$$

The ("mirror") quadrupole

$$q_{x,y,u,v} = \frac{1}{N} tr[S_F(x)\, S_F^\dagger(y)\, S_F(u)\, S_F^\dagger(v)]\,; \qquad \bar{q}_{x,y,u,v} = \frac{1}{N} tr[\bar{S}_F(x)\, \bar{S}_F^\dagger(y)\, \bar{S}_F(u)\, \bar{S}_F^\dagger(v)]\,.$$

No higher multipoles appear if the projectile contains only dipoles!

THE WANNABES

$$w_{x,y,v,u} = \frac{1}{N} tr[S_F(x)\, S_F^\dagger(y)\, \bar{S}_F(u)\, \bar{S}_F^\dagger(v)] = q_{x,y,v,u} + t_{x,y,v,u}$$

Remember that we have to set $\bar{S} = S$ at the end of our computation.

The WANNABES become real multipoles, but only at the end of the calculation.

While Σ^P propagates in Y, the WANNABES behave like independent degrees of freedom.

Still, since t is set to zero in the end, it does not wonder very far from zero throughout evolution and one can set up perturbation theory in t.

THE LARGE N LIMIT

In the large N_C limit all color singlets propagate "independently".

Practically this means for any functional F that is propagated by the Hamiltonian H_3:

$$F_Y[s, \bar{s}, q, \bar{q}, w] = F_0[s(Y), \bar{s}(Y), q(Y), \bar{q}(Y), w(Y)]$$

where all the degrees of freedom: s, $\bar{s}$, q, $\bar{q}$, w satisfy differential equations generated by the corresponding "first quantized" Hamiltonian.

Thus AT LARGE N_C

$$H_3 \;\longrightarrow\; H_s \;+\; H_q \;+\; H_t$$

H_s generates Kovchegov evolution for s.

$$\frac{d}{dY} s(x,y) \;=\; K^{BFKL} \otimes (s \;-\; s\,s)$$

H_q generates linear evolution of q (BKP - like) WHICH IS COUPLED TO s.

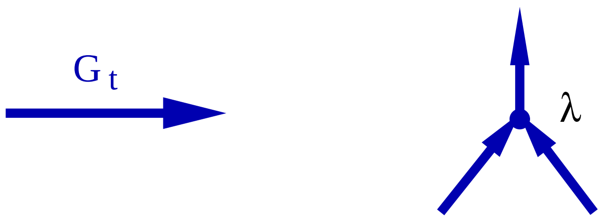
$$\frac{d}{dY}\, q(x,y,u,v) \;=\; K_1 \otimes q \;+\; K_2 \otimes q\,s \;+\; K_3 \otimes s\,s$$

H_t generates a nonlinear evolution of t WHICH IS ALSO COUPLED TO s.

$$\frac{d}{dY}\, t(x,y,u,v) \;=\; G^{-1}[s] \otimes t \;+\; \lambda \otimes t\,t$$

G is a propagator in the external “Pomeron” field s.

For the two point Wannabe (Wannabe dipole) $t(x,y,y,x)$ the propagator is just BFKL: $G \rightarrow G^{BFKL}$.



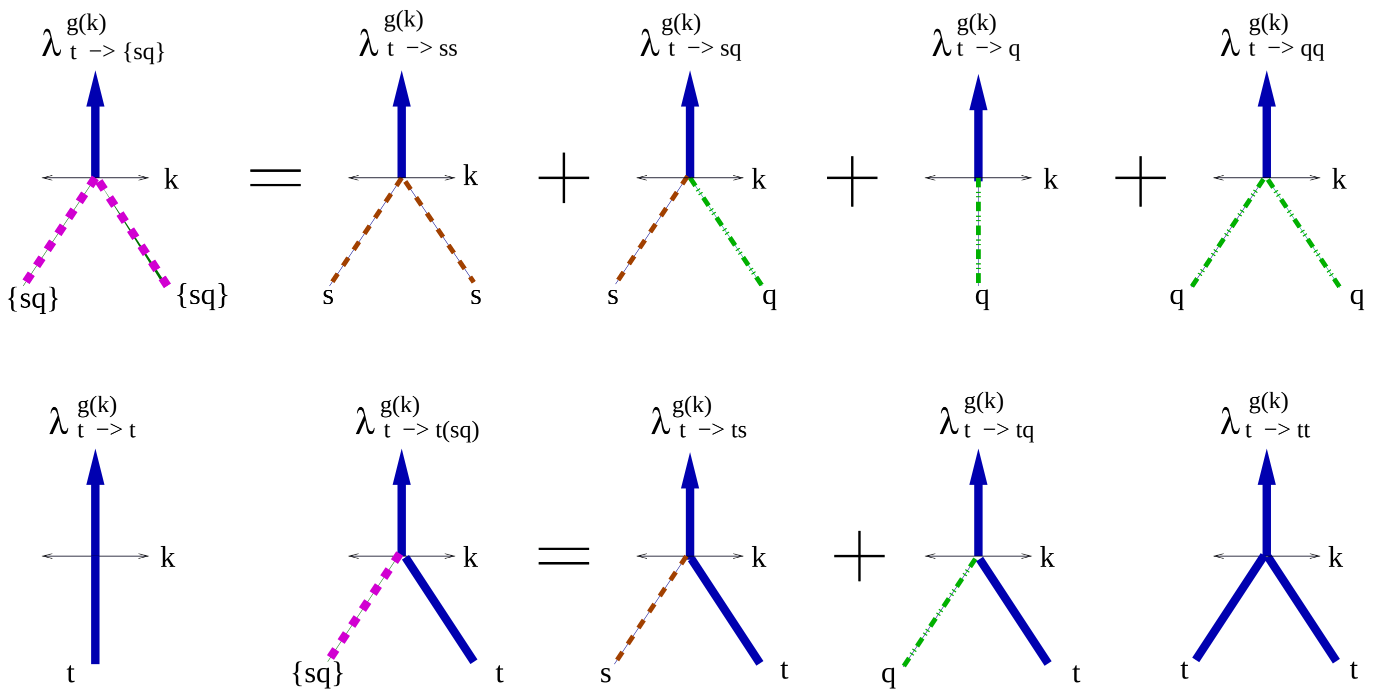
Re-express the insertion operator Q_g

$$\mathcal{O}_g(k) \;=\; A_{-1}(k) \;+\; A_0(k) \;+\; A_1(k)$$

A_{-1} - kills one t

A_0 - propagates t

A_1 - splits one t into two



In the dipole model the projectile is made of dipoles - the the initial condition depends on the Wannabe dipoles:

$$\langle \hat{\mathcal{O}}_n \ldots \hat{\mathcal{O}}_1 \rangle \; = \int Ds\, DqDt\, W^T[s]\, \delta(t)\, e^{-\{H_s+H_q+H_t\}Y_n}\, \mathcal{O}_n[s,q,t]\, U_3(Y_n,\, Y_{n-1}) \times$$

$$\times \;\; \mathcal{O}_{n-1}[s,q,t] \;\ldots e^{-\{H_s+H_q+H_t\}(Y-Y_1)}\, \Sigma_0^{PP}$$

where

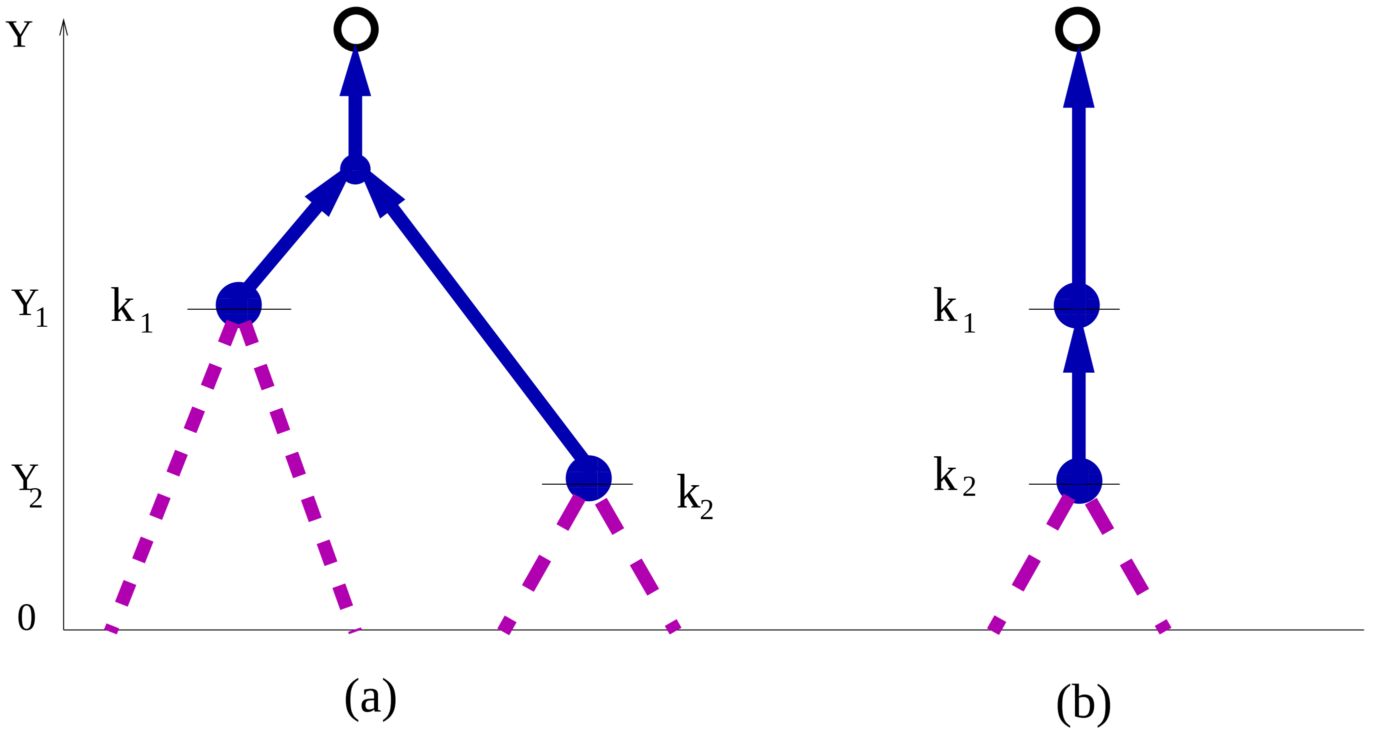
$$\Sigma_0^{PP} = \Sigma_0^{PP}[S^\dagger \bar{S}] = \Sigma_0^{PP}[w_{x,y,y,x}] = \Sigma_0^{PP}[1 + t_{x,y,y,x}]$$

We can only hit n factors of t with derivatives before setting t to zero - at most n insertions of A_{-1}

For any n - finite number of diagramms where the Wannabes propagate from the projectile rapidity to at most the rapidity of the last gluon - and the dipoles and quadrupoles propagate all the way to the target rapidity.

THE WANNABES PROPAGATE IN THE DIPOLE BACKGROUND!

TWO GLUON INCLUSIVE PRODUCTION



J. Jalilian-Marian and Yu. Kovchegov (2004)

Violates AGK cutting rules because the three - and four - point Wannabes couple to dipoles in the propagation.

Should be feasible to calculate numerically...

We can argue for serious simplification in the case of one and two gluon production.

For one gluon - only the propagator of the wannabe dipole appears. This decouples form s and q - is just BFKL

For 2 gluons more complicated since the vertex A_0 turns the two point wannabe into a three point wannabe.

$$A_0 : \qquad t(x,y,y,x) \to \quad t(x,y,y,v)$$

BUT! If the target is saturated with saturation momentum Q_s one can show that $|x-v| < Q_s^{-1}$.

BUT! For small $x - v|$ the propagator of the three point wannabe can be expressed in terms of the wannbe dipole (or just BFKL)

$$G(x_0, y_0, v_0 | x, y, v; \eta) =$$

$$\frac{1}{2}\Big(G^{BFKL}(x_0, y_0 | x, y; \eta) + G^{BFKL}(y_0, v_0 | y, v; \eta) - G^{BFKL}(x_0, v_0 | x, v; \eta)\Big)$$

So life is a bit easier for brave people who would do numerics!

BUT NOT FOR THREE GLUONS AND BEYOND...

THREE GLUONS

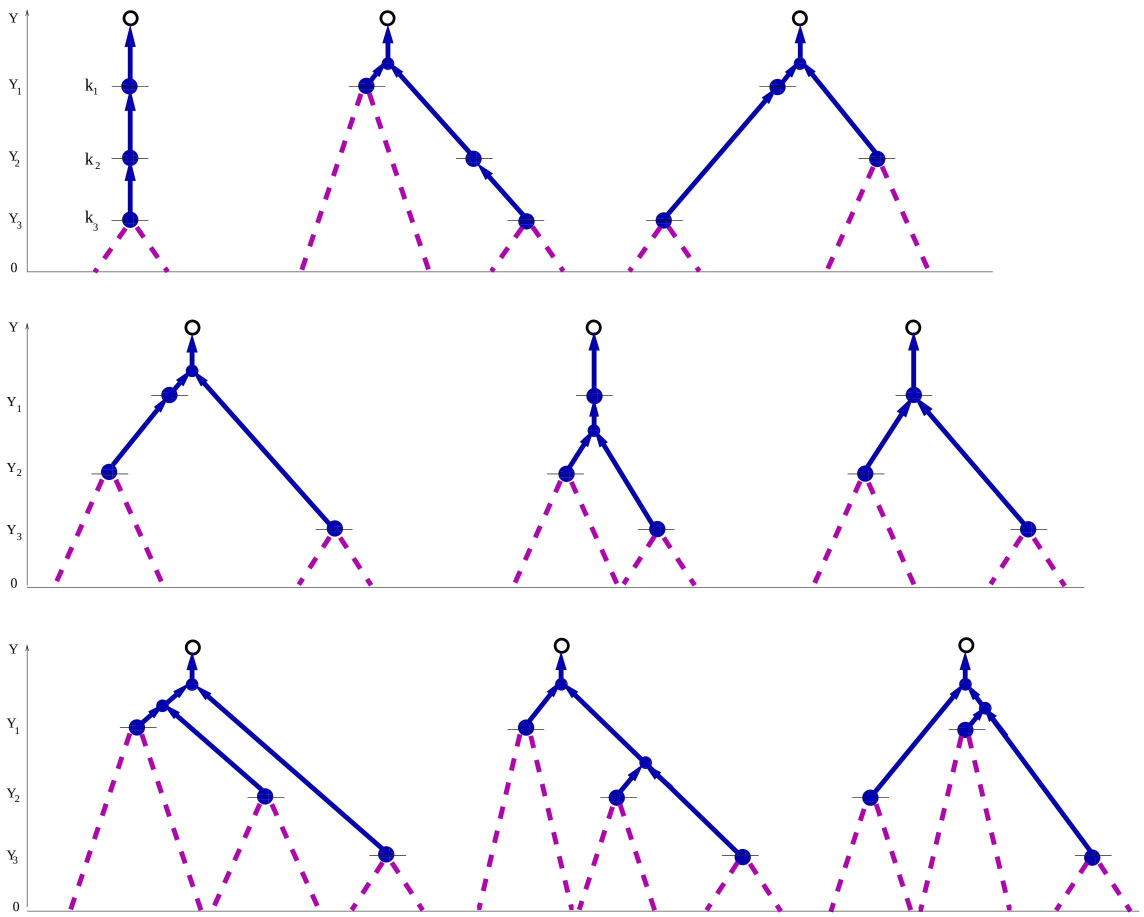


Figure 2: Daniel Kovner and the end of the talk