Multiplicities in Pb-Pb collisions at the LHC from running coupling evolution and RHIC data

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Heavy Ion Collisions at the LHC
Last Call for Predictions

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Balitsky-Kovchegov evolution equation with running coupling

- Recent developments
- Strong reduction of the speed of evolution

Phenomenological consequences:

- Energy dependence of multiplicity densities in A-A collisions
- Determining initial conditions: RHIC @ $\sqrt{s}=130$ and 200 GeV
- Extrapolation to central Pb-Pb collisions @ $\sqrt{s}=5.5$ TeV
• The quark contribution to the BK equation has been calculated recently resumming $\alpha_s N_f$ contributions to all orders, and then completing $N_f \rightarrow -6 \Pi \beta_2$ to determine the scale for the running of the coupling:

\[ \left\langle \begin{array}{c} \tilde{K} \left( r, r_1, r_2 \right) = \frac{N_c}{2\pi^2} \left[ \alpha_s \left( r_1^2 \right) \frac{1}{r_1^2} - 2 \frac{\alpha_s \left( r_1^2 \right) \alpha_s \left( r_2^2 \right)}{\alpha_s \left( R^2 \right)} \frac{r_1 \cdot r_2}{r_1^2 r_2^2} + \alpha_s \left( r_2^2 \right) \frac{1}{r_2^2} \right] \right\rangle \]

• However, the two calculations yield different results:

\[ \left\langle \begin{array}{c} \frac{\partial S (r; Y)}{\partial Y} = \int d^2 z \tilde{K} (r, r_1, r_2) \left[ S (r_1; Y) S (r_2; Y) - S (r; Y) \right] \\
\tilde{K}^{\text{KW}} (r, r_1, r_2) = \frac{N_c}{2\pi^2} \left[ \alpha_s \left( r_1^2 \right) \frac{1}{r_1^2} - 2 \frac{\alpha_s \left( r_1^2 \right) \alpha_s \left( r_2^2 \right)}{\alpha_s \left( R^2 \right)} \frac{r_1 \cdot r_2}{r_1^2 r_2^2} + \alpha_s \left( r_2^2 \right) \frac{1}{r_2^2} \right] \\
\tilde{K}^{\text{Bal}}_{\text{run}} (r, r_1, r_2) = \frac{N_c \alpha_s \left( r_2^2 \right)}{2\pi^2} \left[ \frac{r_1^2}{r_1^2 r_2^2} + \frac{1}{r_1^2} \left( \frac{\alpha_s \left( r_1^2 \right)}{\alpha_s \left( r_2^2 \right)} - 1 \right) + \frac{1}{r_2^2} \left( \frac{\alpha_s \left( r_2^2 \right)}{\alpha_s \left( r_1^2 \right)} - 1 \right) \right] \end{array} \right\rangle \]
@ **Why?**: The inclusion of all orders $\alpha_s N_f$ contributions brings in new physical channels that modify the interaction structure of the equation:

\[
\frac{\partial S(x, y, ; Y)}{\partial Y} = \mathcal{R}[S] - S[S]
\]

- **“Running”** term:
  \[
  \mathcal{R}[S] = \int d^2 z \tilde{K}^{run}(x_0, x_1, z) [S(x_0, z; Y) S(z, x_1; Y) - S(x_0, x_1; Y)]
  \]

- **“Subtraction”** term:
  \[
  S[S] = \int d^2 z_1 d^2 z_2 K^{sub} [S(x_0, w, Y) S(w, x_1, Y) - S(x_0, z_1, Y) S(z_2, x_1, Y)]
  \]

Once the two terms are included the two calculations agree with each other!!
The extra “subtraction” term is numerically important and considerably reduces the speed of the evolution:

- Speed reduction due to subtraction term:
  \( \sim 30\% \) w.r.t. only running in KW’s scheme
  \( \sim 10\% \) w.r.t. only running in Balitsky’s scheme

Caution!!: A particular definition of \( Q_s \)

\[ N\left(r = 1/Q_s; Y\right) = 0.5 \]
The energy dependence of the saturation scale from running coupling evolution is milder than the one extracted from fits to HERA DIS data.

- Fits to HERA: $Q_s^2(x) = Q_0^2 \left( \frac{x_0}{x} \right)^\lambda$; $\lambda \approx 0.288$  
  Golec-Biernat Wüsthoff (98)

- Energy dependence of multiplicity in saturation models for particle production:

  $\frac{dN_{AA}}{dy} \bigg|_{\eta=0} \sim \sqrt{s}^\lambda$

  Kharzeev-Levin-Nardi  
  Armesto-Salgado-Wiedemann (05)  

\[ \lambda = \frac{d \ln Q_s^2(Y)}{dY} \]

- CGC + hydrodynamics at RHIC favours $\lambda=0.2$  
  Hirano-Nara (04)
Particle production in A-A collisions:

\[ \frac{dN_{AA}}{d\eta} \propto \frac{4\pi N_c}{N_c^2 - 1} \int^{p_m} d^2p_t \int^p d^2k_t \alpha_s(Q) \varphi_A \left( x_1; \frac{|p_t + k_t|}{2} \right) \varphi_A \left( x_2; \frac{|p_t - k_t|}{2} \right) \]

- \(2 \rightarrow 1\) kinematics

\[ x_{1(2)} = \frac{p_t}{\sqrt{s}} e^{\pm y} \quad \text{or} \quad x_{1(2)} = \frac{m_t}{\sqrt{s}} e^{\pm y} \]

- Rapidity ↔ Pseudorapidity: Gluon mass

\[ y(\eta, p_t, m) = \frac{1}{2} \ln \left[ \frac{\sqrt{\frac{m^2 + p_t^2}{p_t^2} + \sinh^2 \eta + \sinh \eta}}{\sqrt{\frac{m^2 + p_t^2}{p_t^2} + \sinh^2 \eta - \sinh \eta}} \right] \]

- Running coupling: \( Q = \max \left\{ \frac{|p_t \pm k_t|}{2} \right\} \)

\[ \varphi(x, k) \Rightarrow \text{Solutions of BK with running coupling} \times (1 - x)^4 \]

\[ \varphi(x, k) = \int \frac{d^2r}{2\pi^2 r^2} \exp^{ik \cdot r} \mathcal{N}(x, r) \]

Local Hadron Parton Duality
@ Initial conditions for evolution: Au-Au central collisions at RHIC
at $\sqrt{s} = 130$ and 200 GeV

- McLerran-Venugopalan i.c. $N_A(r, Y_{ev} = 0) = 1 - \exp \left\{ -\frac{r^2 Q_0^2}{4} \ln \left( \frac{1}{|r\Lambda|} + e \right) \right\}$

\[
\varphi(x, k) = \int \frac{d^2 r}{2\pi^2 r^2} e^{ik \cdot r} N(x, r)
\]

**Things to fix:**

- effective gluon mass, $m$
- Initial saturation scale $Q_0$
- Is there significant evolution prior to $\sqrt{s} = 130$?
Initial conditions for evolution:
Au-Au central collisions at RHIC at $\sqrt{s} = 130$

Gluon mass $\sim 0.25$ GeV

Initial saturation scale $Q_0 \sim 1$ GeV

PHOBOS data
Is there significant evolution prior to $\sqrt{s} = 130$ at central rapidity?: NO!

\[ Y = \ln \left( \frac{x_0}{x} \right) + \Delta Y_{ev}, \quad x_0 = 0.1, \quad x(\eta = 0) = \frac{p_t}{\sqrt{s}} \]

- RHIC energies are governed by pre-asymptotics effects (MV model: good i.c.)
- Solutions close to the scaling region fail to reproduce RHIC data: No universality

\[ \lambda = \frac{d \ln Q_s^2(Y)}{dY} \frac{dN_{ch}^{\text{ch}}}{d\eta} \]

PHOBOS data
Very good agreement with RHIC data with:

⇒ Gluon mass: \( m = 0.2 \div 0.3 \) GeV

⇒ Initial saturation scale: \( Q_s(\sqrt{s}=130 \text{ GeV}, \eta=0) = 0.9 \div 1.1 \) GeV

⇒ Pre-asymptotic regime: \( \Delta Y_{ev} \leq 2 \)
Extrapolation to LHC Pb-Pb central collisions at $\sqrt{s}=5.5$ TeV

$\frac{dN^{ch}}{d\eta}$

$\sqrt{s_{NN}}=5.5$ TeV

$\approx 2100 \div 1800$

$\approx 1700$

$\approx 1400$

$\approx 1100$

KLN

NPA 747
620-629 (05)

ASW

hep-ph/0703146

PHOBOS data

Extrapolation to LHC Pb-Pb central collisions at $\sqrt{s}=5.5$ TeV

$\sqrt{s_{NN}}=200$ GeV

$\sqrt{s_{NN}}=130$ GeV

W Busza
APP B35 2803(04)
@ Au-Au data at RHIC energies is compatible with both logarithmic and power-law behaviour with respect to collision energy.

@ Logarithmic trend seems to be dictated from lower energies data.
@ Higher order corrections considerably reduce the speed of non-linear evolution

@ Multiplicity densities at RHIC can be reproduced using \( \text{kt-factorization} + \text{solutions} \) of the evolution

\[ \Rightarrow \text{gluon mass} \approx 0.2 \div 0.3 \text{ GeV} \]

\[ \Rightarrow Q_s(\sqrt{s}=130 \text{ GeV}, \eta=0) \approx 0.9 \div 1.1 \text{ GeV} \]

\[ \Rightarrow \text{Pre-asymptotic regime: strong scaling violations} \]

@ Extrapolation to Pb-Pb central collisions at \( \sqrt{s}=5.5 \text{ TeV} \) yields a central value:

\[ \left. \frac{dN_{\text{evol}}}{d\eta} \right|_{\eta=0} (\sqrt{s} = 5.5 \text{ TeV}) \approx 1400 \]

@ Smaller than predictions based on HERA information

\[ \left. \frac{dN^{\lambda=0.288}}{d\eta} \right|_{\eta=0} (\sqrt{s} = 5.5 \text{ TeV}) \approx 2100 \div 1700 \]

@ Larger than empiric extrapolations from lower energies data

\[ \left. \frac{dN^{\text{log ext}}}{d\eta} \right|_{\eta=0} (\sqrt{s} = 5.5 \text{ TeV}) \approx 1100 \]
What’s next?

@ Evolution equation:

⇒ **Gluon contribution** to high order corrections
⇒ Beyond mean field: **Pomeron loops, fluctuations**
⇒ Impact parameter dependence
⇒ Energy conservation

They all point to an even stronger reduction of the speed of evolution!!

@ Particle Production:

⇒ **Factorization breaking** terms (Classical YM EOM?)
⇒ NLO calculation
⇒ Large-\(x\) effects
⇒ Proper inclusion of **non-perturbative effects** (CGC + Hydro?)
⇒ Better knowledge of **pre-equilibrium / thermalization** dynamics
Back up slides
pt vs mt

Extrapolation at $\eta=0$
unaffected

differences only in the forward region
Once the subtraction term is added back, the two approaches agree:
• The separation procedure is similar in both calculations:

\[ S[S] = \int d^2z_1 \, d^2z_2 \, K^{sub} \left[ S(x_0, w, Y) \, S(w, x_1, Y) - S(x_0, z_1, Y) \, S(z_2, x_1, Y) \right] \]

• The differences between the two approaches stem from the choice of the subtraction point, w
• In Balitsky’s scheme: $w = z_1$ (or $z_2$), the quark’s (anti-q) transverse position:

$$S^{Bal}[S] = \int d^2 z_1 d^2 z_2 K^{sub} [S(x_0, z_1, Y) S(z_1, x_1, Y) - S(x_0, \tilde{z}_1, Y) S(\tilde{z}_2, x_1, Y)]$$

An expansion in term of $N$’s result in just non-linear terms ($N^2<<N$ at small-$r$)

• In KW scheme: $w = z =$, the gluon’s transverse position:

$$S^{KW}[S] = \int d^2 z_1 d^2 z_2 K^{sub} [S(x_0, z, Y) S(z, x_1, Y) - S(x_0, \tilde{z}_1, Y) S(\tilde{z}_2, x_1, Y)]$$

An expansion in term of $N$’s also includes linear terms.

• The kernel of the subtraction contribution is the same in both cases:

Leading-$N_f$ $\Rightarrow$ All orders $N_f$ (LCPT) $\Rightarrow$ Fourier transform to coordinate space
• Here: $N_f \rightarrow -6 \Pi \beta_2$. Part of the gluon contribution is also taken into account.
• The subtraction term is larger in KW’s scheme than in Balitsky’s:

\[ \mathcal{F} = \mathcal{R} - \mathcal{S} \]
The subtraction term is larger in KW’s scheme than in Balitsky’s:

The relative contribution of the subtraction term to the evolution fades away at large rapidity.