Testing statistical hadronization with $\nu_{K/\pi}$ (And others)

Giorgio Torrieri
What we know... (Kaneta and Xu, nucl-th/0405068, also Braun-Munzinger, Stachel, Becattini, Rafelski, GT,...)

This will probably also happen at the LHC!
...But it also happens in $e^+e^-, p-p!$ (Becattini, hep-ph/0108212)
What does this mean?

• Kinetic thermalization? String breaking? Black holes? Phase space?

• Is the cause for thermalization the same in p-p and Au-Au?

• **which** statistical model?
  
  – **Canonical** suppression to model strange particles at low energies/small systems?
  – Is $\gamma_s$ needed? (**Chemical** under-saturation vs enhancement in QGP phase)
  – Is $\gamma_q$ needed? (thermal **coalescence** of existing quark flavor. Low $S/V$ at low energy/system size, high $S/V$ in A-A)

What are the implications of all this at the LHC? Does this have **phenomenological** consequences?
How do you falsify statistical models?

This:

Could be as useful as

for statistical models

for Inflationary models

Lets use fluctuations (of \( \pi, K, p, \ldots \)) NOT to look for new physics but to constrain/rule out existing models

How do the parameters describing yields/ratios describe fluctuations?
Is there a universal freeze-out volume? (statistics needs it!)

\[ \langle (\Delta N)^2 \rangle = \langle (\Delta \rho)^2 \rangle \langle V \rangle + \langle (\Delta V)^2 \rangle \langle \rho \rangle \]

\textbf{Statistical Centrality (Understood, requires, correcting)}

"Dynamical" Not understood (KNO?)

\textbf{NB:} Fluctuations in \( e^+e^- \) seem thermal (Poisson), but \( p-p \) do not (KNO scaling)

- unless (maybe!) \( \langle (\Delta V)^2 \rangle \sim \langle V \rangle \) (Pressure ensemble)

- Strings also reproduce KNO (K. Werner, PRL 61:1050, 1988)
Solution:

Use fluctuations of ratios, volume fluctuation $\Delta V$ cancels out e-by-e

$$\sigma_{N_1/N_2}^2 = \langle \left( \frac{\Delta N_1}{N_1} - \frac{\Delta N_2}{N_2} \right)^2 \rangle$$

$\langle (\Delta V)^2 \rangle$ cancels out between $\frac{\Delta N_1}{N_1}$ and $\frac{\Delta N_2}{N_2}$ but

$$\sigma_{N_1/N_2}^2 \sim \frac{1}{\langle V \rangle}$$

$\langle V \rangle \sim \langle N \rangle$ the same as for multiplicities. NOT guaranteed kinetic, string and other non-equilibrium models will give the same scaling.
Get rid of volume dependence by using

\[ \frac{d\langle N_1 \rangle}{dy} \sigma_{N_1/N_2}^2 \]

If uncorrelated independent sources such as the Grand Canonical Ensemble (or HIJING!)

\[ \text{Statistical Model Parameters} \]
**IF** the chemical conditions are $\simeq$ the same (such as RHIC $\rightarrow$ LHC, provided freeze-out at equilibrium) $\frac{d\langle N_\pi \rangle}{dy} \sigma^2_{K/\pi,\pi^+/\pi^-,K^+/K^-,p/\pi}$ should stay CONSTANT with $\frac{\langle dN_\pi \rangle}{dy}$ across energies.

**IF** $\gamma_q, \gamma_s$ jump at some critical energy/system size, so should $\frac{d\langle N_\pi \rangle}{dy} \sigma^2$. (Quantum corrections bigger for $\sigma^2_N$ than $\langle N \rangle$)

$T - \gamma$ **correlate** for yields, **anti-correlate** for fluctuations. Describe both!
Global correlations (E.G. Canonical ensemble) spoil this scaling

If $N_1$ is canonical...

"Wiggle"


For $\frac{d\langle N_\pi^-\rangle}{dy} \sigma^2 K^+ / K^-$ discrepancy is by a factor of $\frac{1}{2}$, for $\frac{d\langle N_\pi^-\rangle}{dy} \sigma^2 K^\pm / \pi^\pm$ less
Detector cuts result in an additional contribution to fluctuations, that needs to be subtracted from “physics.”

Same fluctuations are evident in mixed events, so use

\[ \sigma_{dyn}^2 = \sigma^2 - \sigma_{mixed}^2 \approx \sigma^2 - \frac{1}{N_1} - \frac{1}{N_2} \]

Cuts effect on correlations (due to resonances in hadron gas)

\[ \langle \Delta N_1 \Delta N_2 \rangle \sim \langle N^* \Rightarrow N_1 N_2 \rangle \]

more complicated. Needs to be simulated by the same algorithms used to correct for cuts in resonance yield measurements.

Deviations from scaling (Wiggle,...) should not be affected, provided ratios weakly correlated by resonances \((K^+ / \pi^+, K^+ / K^-, ...)\) used
To identify wiggle one needs to go to very low centrality events, where $N_{1,2}$ could be 0 and $N_1/N_2$ acquires very high higher cumulants.

\[ \nu_{N_1/N_2}^{dyn} = \frac{\langle N_1(N_1 - 1) \rangle}{\langle N_1 \rangle^2} + \frac{\langle N_2(N_2 - 1) \rangle}{\langle N_2 \rangle^2} - \frac{\langle N_1 N_2 \rangle}{\langle N_1 \rangle \langle N_2 \rangle} \]

where \( \langle ... \rangle \) refers to averaging over all events

"Theoretically" \( \nu_{N_1/N_2}^{dyn} = (\sigma_{N_1/N_2}^{dyn})^2 \)

"Experimentally" It is measured very differently, histogramming over all events. 2nd cumulant isolated even in low multiplicity events.

Can go to low centralities and really explore how \( \nu_{N_1/N_2} \frac{d\langle N_1 \rangle}{dy} \) scales
What will happen at the LHC? Well, if you believe that...

**Hadronization happens in equilibrium**
Difference between RHIC and LHC very small

**Strangeness is canonical in RHIC acceptance**
Same as above, but $\frac{d\langle N^- \rangle}{dy} \nu_{K/\pi,K^+/K^-}$ shows kink at low multiplicity

**Hadronization at the LHC does NOT happen in equilibrium**
LHC $\frac{d\langle N^- \rangle}{dy} \nu_{K/\pi,K^+/K^-}$ still flat at LHC, but greater $\gamma_{s,q}$ ensures increase
of $\frac{d\langle N^- \rangle}{dy} \nu_{K/\pi,K^+/K^-}$ w.r.t. RHIC

**Mini (and not so mini) jets dominant** (Non statistical)
Scaling of $\frac{d\langle N^- \rangle}{dy} \nu_{K/\pi}$ w.r.t. multiplicity most likely broken
Results (parameters from Rafelski and Letessier, EPJ.C45:61-72, 2006.)

Equilibrium (T=156, $\gamma_q=\gamma_s=1$)

Non-eq. 1 (T=145, $\gamma_q=\gamma_s=1.62$)

Non-eq. 2 (T=134, $\gamma_q=1.67, \gamma_s=3$)

Non-eq. 3 (T=125, $\gamma_q=1.73, \gamma_s=5$)
The effect of quantum fluctuations at high $\gamma_{q,s}$

![Graph showing the effect of quantum fluctuations at high $\gamma_{q,s}$](image_url)
Comparison with RHIC results

RhIC (Matches $T=140$ MeV, $\gamma_q =1.6, \gamma_s =2$)

Higher $\gamma_{q,s}$

Equilibrium ($T=156, \gamma_q =\gamma_s =1$)

Non-eq. 1 ($T=145, \gamma_q =1.62$)

Non-eq. 2 ($T=134, \gamma_q =1.67, \gamma_s =3$)

Non-eq 3 ($T=125, \gamma_q =1.73, \gamma_s =5$)
Canonical suppression of strangeness and $\frac{dN_{\pi}}{dy}\nu K^+ / K^-$

- Equilibrium (T=156, $\gamma_q=\gamma_s=1$)
- Non-eq. 1 (T=145, $\gamma_q=\gamma_s=1.62$)
- Non-eq. 2 (T=134, $\gamma_q=1.67, \gamma_s=3$)
- Non-eq. 3 (T=125, $\gamma_q=1.73, \gamma_s=5$)
- Equilibrium Canonical

$\sim dN_K / dy = 10$
• Test statistical model by how fluctuations scale w.r.t. yields

\[ \frac{d\langle N_1 \rangle}{dy} \nu_{N_1/N_2} \] nice scaling variable as

- Should be flat w.r.t. \( \frac{d\langle N_1 \rangle}{dy} \) in simplest statistical model
- Unless \( T, \mu, \gamma \) changes, no variation across energy
  The absolute value is highly sensitive to chemical non-equilibrium
- More complicated models (Canonical effects, admixture from non-thermal sources) generally give observable deviations
- Is stable against cuts

Conclusion:
Could be as useful as
for Inflationary models
for statistical models
Conclusion:
Could be as useful as

for statistical models

This:

for Inflationary models

How many \( \frac{d\langle N_1 \rangle}{dy} \nu_{N_1/N_2}^{dyn} \) are described by \( T, \mu, \gamma_{q,s}, \) volume you use for yields?
Do they scale the right way with energy/centrality?