Jet Evolution in the Quark Gluon Plasma

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Outline of the talk:

- Jet evolution in vacuum in a leading log picture
- Modification of the evolution equations in the quark gluon plasma
- Evolution of the multiplicity, centroid and width of the $dN/d \log x$ distribution of partons in the jet
- Speculations on further work
Jets become important at LHC

$10^{11}$/month

Figure 6.2: Number of jets with $E_T > E_T^{min}$ and $|\eta| < 0.5$ produced in Pb–Pb collisions in one effective month of running ($10^9$ s). The minimum bias rate (solid line) is compared to the rate in 10% most central collisions (dashed line).

Figure 6.3: Average number of jets with $E_T > E_T^{min}$ and $|\eta| < 0.5$ per event in the 10% most central Pb–Pb collisions.
Parton evolution by branching processes

- We use a distribution function which has three variables:
  - Momentum fraction $z$, the virtuality (mass squared) $Q^2$ and the transverse momentum $p_T^2$.

\[
D^j_i(z, Q_0^2, \vec{p}_t) = \delta(i, j)\delta(x-1)\delta^2(p_T)
\]

(18)

and the equation is:

\[
Q^2 \frac{\partial D^j_i(z, Q^2, \vec{p}_t)}{\partial Q^2} =
\]

\[
= \frac{\alpha_s(Q^2)}{2\pi} \int_{z}^{1} \frac{du}{u} P^r_i(u, \alpha_s(Q^2)) \frac{d^2 q_t}{\pi} \delta(u(1-u)Q^2 - Q_0^2/4 - q_t^2) D^j_i(z/u, Q^2, \vec{p}_t - z/u q_t)
\]

(19)
Solution of evolution in vacuum

Next we define the Mellin Transform of the equation:

\[ d(J, Q^2) = \int_0^1 dz D(z, Q^2) z^{J-1} \]

\[ d(J, Q^2) = c \left( \frac{Q^2}{Q_0^2} \right)^{\gamma(J, \alpha_s)} \]

\[ c \exp[\gamma \log(Q^2/Q_0^2)] = c \exp[\int_{Q_0^2}^{Q^2} dQ'^2 \gamma(J, \alpha_s)/Q'^2] \]

- First, we use the $p_t$ integrated distribution, and go over to the Mellin Transform
- Then, we make an ansatz with an anomalous dimension $\gamma (\alpha_s,)$
- $\gamma (\alpha_s)$ obeys a quadratic equation
Concentrate on small $z$-behavior

\[ \gamma(J, \alpha) = \]
\[ = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 du 2C_A u^{J-2+2\gamma} \]

\[ n(Q^2) = c \exp\left[ \frac{1}{b} \sqrt{\frac{2C_A}{\alpha_s \pi}} \right] \]
\[ b = \frac{11}{4\pi} \]

- Evolution is limited to the dominant gluon-gluon splitting $P(u) = c/u$
- Suppression of soft gluon emission due to coherence, which is reflected in the exponent
- Resulting multiplicity $n(Q)$ in vacuum after integration over $Q^2$ or $\alpha_s(Q^2)$
- Multiplicity increases with virtuality
Differential multiplicity distribution $dN/d \ln(1/x)$ in a jet of $(90 \text{ GeV})^2$
What happens in the quark gluon plasma?

- The parton can make collisions with the plasma particles.
- The density of plasma particles $n_g = 2T^3$.
- The cross section is determined by Debye-screened gluon exchange.
- The lifetime for a collision is estimated $t = E/Q_0^2$, where we use as an upper bound the $Q^2$ of the dominant lowest virtuality gluons. $Q_0 = 2$ GeV is the infrared cutoff.
Modification of evolution equation in the quark gluon plasma

\[ Q^2 \frac{\partial D^j_i(z, Q^2, \vec{p}_t)}{\partial Q^2} = \]

\[ = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u} \frac{d^2\vec{q}_t}{\pi} P_r(u, \alpha_s(Q^2)) \delta(u(1-u)Q^2 - Q_0^2 - q_t^2)D^j_i(z/u, Q^2, \vec{p}_t - z/u\vec{q}_t) + S(Q^2, \vec{p}_t) \]

(23)

with the scattering term $S(Q^2, p_t)$, note The Lorentz factor is really $zE$.

\[ \tilde{s}(Q^2, \vec{p}_t) = zE/Q_0^2 n_g \int_z^1 dw \int d^2\vec{q}_t \frac{d^2\sigma^r_i}{d^2\vec{q}_t} (D^j_i(w, Q^2, \vec{p}_t - w\vec{q}_t) - D^j_i(z, Q^2, \vec{p}_t)) \delta(w - z - \frac{q_t^2}{2m_gE}). \]

(24)

As before, we limit ourselves to the gluon cascade
Method of solution is similar to the vacuum case

- The scattering term shows up as a drift term to smaller $z$ value

\[
\hat{S}(Q^2) = \frac{zE_n g \sigma_i^r}{Q_0^2 2m_g E} \frac{\partial D_i^r(z, Q^2)}{\partial z} 
\]

We read off a small parameter $\delta$, which is scale invariant, i.e. no longer dependent on the energy and defined as:

\[
\delta = \frac{z n_g \sigma_i^r < p_t^2 >}{2 m_g Q_0^2} \tag{29}
\]

\[
\sigma_g^g(Q^2, T) = \frac{9 \pi \alpha(Q^2)^2}{2 m_D^2} \tag{30}
\]

The mean transverse momentum squared I assume is near the Debye mass squared.

\[
<p_t^2> \approx m_D^2 \tag{31}
\]
Subleading correction to the DGLAP evolution

- The scattering term $S$ leads to a subleading correction to the dominant $\alpha_s$ evolution. The medium modification term is proportional to $\alpha_s^{(3/2)}$.
- Its temperature dependence is $T^2$.
Mean Multiplicity in the Quark Gluon Plasma at temperature $T$

- Mean multiplicity is growing with increasing temperature for $T=0.8$ GeV and $T=1.0$ GeV
- The total multiplicity in vacuum is fixed with $e^+e^-$ data
- The medium multiplicity at the starting scale of evolution, i.e. in the infrared, is fixed to be equal to the vacuum value
Centroid of the Gaussian Distribution in $\ln(1/x)$

Figure 2: Centroid of the $\ln(1/x)$ distribution in vacuum (lowest curve), at $T = 0.8\text{GeV}$ (middle curve) and at $T = 1.0\text{GeV}$ (top curve)
Width of the in-medium $\ln(1/x)$ distribution

Figure 3: Width of two jet $\ln(1/x)$ distribution with invariant mass $Q^2$ in vacuum (top curve), at $T = 0.8 GeV$ (middle curve) and at $T = 1.0 GeV$ (lowest curve)
Jet particle spectrum in vacuum and in the quark gluon plasma

Figure 5: Differential multiplicity $dN/d\ln(1/x)$ of jet particles inside a jet with invariant mass $Q^2 = 90 GeV^2$ in vacuum (fine drawn curve), at $T = 1.0 GeV$ (full curve)
Conclusions

• The leading log approximation gives a qualitative picture of jet evolution in the plasma
• Results of evolution equations show a higher multiplicity shifted to lower $x$ with a narrower distribution
• They are qualitatively similar to the results of Borghi and Wiedemann (Nucl. Phys. A 774 (2006) 540), but they can be calculated parametric as a function of plasma density or temperature
• With this calculation the jet measurement becomes a tool of diagnosis for the quark gluon plasma
• Necessary to calculate the Monte Carlo cascade in fragmentation together with collisions (K. Zapp, S. Domdey)
• Challenging new questions regarding the input cross sections and medium effects.
Method of full solution:

\[
d(J, Q^2) = c \left( \frac{Q^2}{Q_0^2} \right)^\gamma(J, \alpha) = \exp[\gamma \log(Q^2/Q_0^2)] = \exp[\int_{Q_0^2}^{Q^2} dQ^2 \gamma(J, \alpha)/Q^2]
\]

• Make a Taylor expansion of the anomalous moment around \( J=1 \), the dominant Mellin amplitude.

• The inverse Mellin transform from the integration in the complex \( J \) plane along the imaginary axis gives a Gaussian multiplicity distribution in \( \ln(1/x) \)

• The zeroth order term is the normalization, the linear term gives the centroid and the quadratic the width of the Gaussian distribution in \( \ln (1/x) \)
High $p_T$-suppression, due to parton energy loss?

Fig. 36. $\pi^0 R_{AA}(p_T)$ for central (0–10 %) and peripheral (80–92 %) Au+Au collisions
Space time development (Initial virtuality $t_0=100 \text{ GeV}^2 \rightarrow t_1$)

Take RHIC case:
Mean final virtuality of radiated gluons is $t_1=10 \text{ GeV}^2$

Mean time for radiation
$<t>=0.7 \text{ fm/c}$
Space time Structure of hadron production

- Induced radiation and fragmentation can not be separated
- The produced parton has time like virtuality and loses energy even in vacuum (vacuum energy loss).
- No difference in decay time between charm quarks and light quarks, because of high initial virtuality
- Most descriptions treat first the energy loss of an on shell quark in the medium and then hadronization, as below

$$z_c D'_{h/c}(z_c, Q^2_c) = z'_c D_{h/c}(z'_c, Q^2_c) + N_g z_g D_{h/g}(z_g, Q^2_g);$$

$$z'_c = \frac{p_h}{p_c - \Delta E_c(p_c, \phi)}, \quad z_g = \frac{p_h}{\Delta E_c(p_c, \phi)/N_g},$$

Modification of fragmentation function separated from energy loss is not justified
Mean $p_t^2$ of the jets

- With the help of the triple differential distribution very detailed questions can be analyzed.
- Increase from $p_t^2 = 0$ at infrared scale $Q_0^2 = 4$ GeV$^2$.
- Top curve is for $T=0$, middle $T=0.2$ GeV and bottom for $T=0.4$ GeV.
- An improved method will use Monte Carlo Methods to get solutions, instead of using the Gaussian parametrisation.
Jet quenching $q$ in a Wilson Line
Calculation comes out too small

- Calculation of Antonov, Dietrich and HJP
- $q<1 \text{ GeV}^2 /\text{fm}$ for $T<1 \text{ GeV}$

$$\langle \mathcal{W}_{L_{||} \times L_{\perp}}^{\text{adj.}} \rangle = \exp \left( -\frac{\hat{g}}{4\sqrt{2}} L_{||} L_{\perp}^2 \right).$$
Medium induced scattering

- Mean free path is shorter due to larger coupling $\alpha(k, T)$
- Debye mass can be determined self-consistently from strong coupling $\alpha(k, T)$
- Running $\alpha(k, T)$ at finite temperature is calculated from RG equation (J. Braun, H. Gies, hep-ph/0512085)

\[
d\sigma_i/dq_{\perp i}^2 \approx C_i \frac{4\pi \alpha^2}{(q_{\perp i}^2 + \mu^2)^2}
\]
Plan: Try to calculate Wilson loop with field strength correlators

\[ < W[C] > = e^{-\sigma R_0 \alpha T} \]
\[ \sigma = \frac{\pi^3 G_2 a^2 \kappa}{18}, \]
\[ \alpha^2 = 1 - \cos^2 \phi \sin^2 \theta. \]

\[ \text{Area} = TR_0 \int_{-1/2}^{1/2} du \int_0^1 dv \sqrt{\left( \frac{dX_{\mu}}{du} \right)^2 \left( \frac{dX_{\mu}}{dv} \right)^2 - \left( \frac{dX_{\mu}}{du} \frac{dX_{\mu}}{dv} \right)^2} \]
\[ = TR_0 \alpha. \]
Inverse Mellin Transform

• Integration over J in the complex plane can be done analytically
• Gaussian integral

The inverse Mellin transform is calculated as follows with $\tilde{J} = -i(J - 1)$:

$$D(z, Q^2) = \frac{C}{2\pi i} \int_{1-i\infty}^{1+i\infty} dJ \frac{1}{x^J} \exp[a(J-1)^2-b(J-1)] = \frac{C}{2\pi} \int_{-\infty}^{+\infty} d\tilde{J} \exp[-a\tilde{J}^2 + i\tilde{J}(-b+Ln[1/x])]$$

(41)

The lowest order coefficient of the $J - 1$ expansion is related to the multiplicity, the next coefficient to the center of gravity of the distribution in $Log(1/x)$ of the generated gluons, and the quadratic coefficient to the width of the distribution in $ln(1/x)$.
Energy loss can be related to the expectation value of a Wilson loop in thermal configuration

\[
\frac{1}{N^2 - 1} \langle \langle \text{Tr} \left[ W^A(y) W^A(x) \right] \rangle \rangle_t \approx \exp \left[ -\frac{C_A}{4 C_F} \int d\xi n(\xi) \sigma(x - y) \right] \\
\approx \exp \left[ -\frac{(x - y)^2}{8} \frac{C_A}{C_F} Q_s^2 \right].
\]

The two Wilson lines are given by the quark in the amplitude \( T \) and the quark in the complex conjugate amplitude \( T^* \). This artificial pair forms a dipole which can be handled with standard methods. For a homogeneous medium the integral over the traversed length gives the length \( L^- \), and \( L^2 \) is the dipole size.

\[
\langle W^A(C) \rangle \approx \exp \left[ -\frac{1}{4\sqrt{2}} \hat{q} L^- L^2 \right]
\]