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CHIRAL BOSONS COUPLED TO ABELIAN GAUGE FIELDS

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ABSTRACT

Chiral bosons coupled to abelian gauge fields are considered using Siegel approach to chiral bosonic theories. The quantization of the system is carried out in the Schrödinger representation after using the BRST procedure to deal with Siegel gauge invariance. It is shown that the system contains the same physical degrees of freedom as the corresponding chiral Schwinger model.

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Chiral bosons have received much attention in the last years due to their presence in the construction of many string models. There are two proposals for the Lagrangian formulation of chiral bosons [1,2]. The Siegel action [1] describes chiral bosons through the introduction of an auxiliary-gauge field. This action has a gauge invariance (the so called "Siegel symmetry") which is anomalous at the quantum level. However, this anomaly can be cancelled by the introduction of a local counterterm [3,4]. Furthermore, it was shown in [4] that a BRST quantization of the modified Siegel action leads to chiral bosonic degrees of freedom only if an auxiliary-gauge field is introduced for each couple of bosonic fields. The presence of the anomaly cancelling term in the modified Siegel action has made its consistent coupling to gravity rather difficult. In fact, only $N = 2$ supersymmetric extensions of Siegel action have been coupled consistently to gravity (supergravity) leading to a Lagrangian bosonic formulation of the heterotic string [5]. Other approaches [6,7,8,9] have given only a partial solution to the problem. The other proposal for a Lagrangian formulation of chiral bosons [2] does not make use of auxiliary fields at the price of not having manifest Lorentz invariance. Surprisingly, this formulation seems to have given the simplest path for the coupling of chiral bosons to gravity [10,11] and to supergravity [12].

In this letter we present in the Siegel formulation the quantization of chiral bosons coupled to abelian gauge fields. Our starting action is based on the one proposed in [6]. Upon quantization, we show that this model is equivalent to the chiral Schwinger model [13] with two chiral fermions of the same chirality and arbitrary charges. The chiral Schwinger model has been analyzed from many points of view [14] and recently it has been quantized using the Schrödinger representation [15]. We use this representation in our quantization after dealing with the Siegel symmetry within the BRST formalism.

Before presenting our analysis let us state our notation. We work in a flat two-dimensional Mikowski space with metric $\eta^{\alpha\beta}$. We label our coordinates by σ and τ where σ is spatial and lying on the circle ($0 \leq \sigma < 2\pi$) and τ is temporal. The signature of our metric is such that $\eta^{\sigma\sigma} = -\eta^{\tau\tau} = 1$. Often we will label our coordinates by $\sigma^\pm = \frac{1}{\sqrt{2}}(\sigma + \tau)$, so that $\partial_\pm = \frac{1}{\sqrt{2}}(\partial_\sigma \pm \partial_\tau)$.

Let us consider two bosonic fields $\phi^i(\tau,\sigma)$, $i = 1,2$ with $U(1)$ charges e^i and a vector field $A_\alpha(\tau,\sigma)$. Our starting Lagrangian is the modified version of Siegel action for two bosonic fields [3,4] coupled to the abelian gauge field A_α [6]:

$$\begin{aligned}\mathcal{L} =& -\partial_+\phi^i\partial_-\phi^i + \frac{1}{2}\lambda^{++}(D_+\phi^i)(D_+\phi^i) - \alpha\lambda^{++}\omega^i\partial_+D_+\phi^i \\ &+ 2A_+e^i\partial_-\phi^i + \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{2}ae^2A_+A_-,\end{aligned} \tag{1}$$

where a sum over repeated indices, either latin or greek, is understood. Several remarks regarding this Lagrangian are in order. The field λ^{++} corresponds to the auxiliary-gauge field of the original Siegel action [1]. Notice that classically (*i.e.*, when $\alpha = 0$) the field equation of λ^{++} imposes the $U(1)$ invariant chirality condition on ϕ^i, $D_+\phi^i = 0$, where D_+ is the $U(1)$ covariant derivative, $D_+\phi^i = \partial_+\phi^i - e^iA_+$. The third term of the Lagrangian corresponds to the anomaly-cancelling term found in [3,4]. The factors ω^i are any real numbers such that $w^2 = w^iw^i = w^1w^1 + w^2w^2 = 1$ (see [4]). The factor α will be determined to cancel the anomaly related to the Siegel symmetry. The coupling of the bosons to the gauge field is given by the fourth term in (1). Notice that only the (say) right-handed part of ϕ^i couples to the abelian gauge field. It is simple to show that that the Lagrangian (1) leads to the expected $U(1)$ anomaly [6]. In fact, under the transformations $\bar{\delta}\phi^i = e^i\theta$, $\bar{\delta}A_\alpha = \partial_\alpha\theta$ this Lagrangian leads to the most general form of the anomaly [13]. Notice that the last term in $\mathcal{L}$ (*i.e.*, the mass term) has been included to obtain this general form. In this mass term a is an arbitrary parameter and $e^2 = e^ie^i = e^1e^1 + e^2e^2$.

The action corresponding to the Lagrangian (1) transforms under the Siegel symmetry,

$$\begin{aligned}\delta\phi^i =& \epsilon^+D_+\phi^i + \alpha\omega^i\partial_+\epsilon^+, \\ \delta\lambda^{++} =& 2\partial_-\epsilon^+ - \lambda^{++}\partial_+\epsilon^+ + \epsilon^+\partial_+\lambda^{++},\end{aligned} \tag{2}$$

into

$$\delta S = -\alpha^2\int d^2\sigma\,\lambda^{++}\partial_+\partial_+\partial_+\epsilon^+, \tag{3}$$

where we have used the fact that $\omega^2 = 1$. The form of δS is the right one to cancel the anomaly in the Siegel symmetry [3,4]. The determination of the parameter α will be carried out during the BRST quantization of (1), which is our next task.

To BRST quantize (1) with the symmetry (2) we follow closely the steps in [4]. Let us introduce the anticommuting ghost field c^+ and antighost field b_{++}, as well as the commuting Lagrange multiplier B_{++}. The BRST transformations are obtained from (2):

$$\begin{aligned}\delta^{\mathrm{B}}\phi^i =& \Lambda(c^+D_+\phi^i + \alpha\omega^i\partial_+c^+), \\ \delta^{\mathrm{B}}\lambda^{++} =& \Lambda(2\partial_-c^+ - \lambda^{++}\partial_+c^+ + c^+\partial_+\lambda^{++}), \\ \delta^{\mathrm{B}}c^+ =& \Lambda c^+\partial_+c^+, \\ \delta^{\mathrm{B}}b_{++} =& \Lambda B_{++}, \\ \delta^{\mathrm{B}}B_{++} =& 0,\end{aligned} \tag{4}$$

where Λ is an anticommuting constant parameter. We choose a gauge where $\lambda^{++} = 0$. The BRST Lagrangian is obtained by adding the following term to (1):

$$\begin{aligned}\mathcal{L}_{\mathrm{GF+FP}} =& -\frac{i}{2}\tilde{\delta^{\mathrm{B}}}(b_{++}\lambda^{++}) \\ =& -\frac{i}{2}(B_{++}\lambda^{++} - 2b_{++}\partial_-c^+ + b_{++}\lambda^{++}\partial_+c^+ - b_{++}c^+\partial_+\lambda^{++}),\end{aligned} \tag{5}$$

where $\tilde{\delta^{\mathrm{B}}}$ refers to the BRST transformations (4) without the Λ parameter. After shifting the field B_{++} in such a way that all the λ^{++} terms but $B_{++}\lambda^{++}$ drop from $\mathcal{L} + \mathcal{L}_{\mathrm{GF+FP}}$, and integrating B_{++} and λ^{++} out, we obtain the BRST Lagrangian,

$$\mathcal{L}_{\mathrm{q}} = -\partial_+\phi^i\partial_-\phi^i + ib_{++}\partial_-c^+ + 2A_+e^i\partial_-\phi^i + \frac{1}{4}F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{2}ae^2A_+A_-. \tag{6}$$

The corresponding conserved BRST current associated to the BRST symmetry turns out to be,

$$J_+^B = c^+\big((D_+\phi^i)(D_+\phi^i) - 2\alpha\omega^i\partial_+D_+\phi^i + ib_{++}\partial_+c^+ + \frac{i}{2}\partial_+b_{++}c^+\big). \qquad (7)$$

Our next step is to carry out the quantization of $\mathcal{L}_{\rm q}$ in the Schrödinger representation. Operators will be time-independent and all time-dependence of the theory will be left in the states. The canonical commutation and anticommutation relations are,

$$\begin{aligned}[\phi^i(\sigma), \pi_\phi^j(\sigma')] =& i\delta(\sigma-\sigma')\delta^{ij}, \\ [A_\sigma(\sigma), E(\sigma')] =& i\delta(\sigma-\sigma'), \\ \{-\frac{1}{\sqrt{2}}b_{++}(\sigma), c^+(\sigma')\} =& \delta(\sigma-\sigma'),\end{aligned} \qquad (8)$$

where,

$$\begin{aligned}\pi_\phi^i =& \partial_\tau\phi^i - \sqrt{2}e^iA_+, \\ E =& \partial_\tau A_\sigma - \partial_\sigma A_\tau.\end{aligned} \qquad (9)$$

The Hamiltonian corresponding to the Lagrangian (6) is:

$$\begin{aligned}H = \int_0^{2\pi} d\sigma\,\Big(&\frac{1}{2}\pi_\phi^i\pi_\phi^i + \frac{1}{2}(\partial_\sigma\phi^i)(\partial_\sigma\phi^i) - e^i(A_\sigma + A_\tau)(\partial_\sigma\phi^i - \pi_\phi^i) + \frac{1}{2}E^2 + E\partial_\sigma A_\tau \\ &+ \frac{1}{2}e^2\big((1-a)A_\tau^2 + 2A_\tau A_\sigma + (1+a)A_\sigma^2\big) - \frac{i}{\sqrt{2}}b_{++}\partial_\sigma c^+\Big).\end{aligned} \qquad (10)$$

Notice that the component A_τ of the vector field does not have canonical momentum and so, since there is not $U(1)$ gauge symmetry in our system, it can be regarded just as a Lagrange multiplier.

We define the components of the Fourier expansions of our fields in the following way:

$$\begin{aligned}\phi^i(\sigma) =& \frac{1}{\sqrt{2\pi}}\Big(\phi_0^i + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{1}{n}(\alpha_n^i{\rm e}^{in\sigma} + \tilde{\alpha}_n^i{\rm e}^{-in\sigma})\Big), \\ \pi_\phi^i(\sigma) =& \frac{1}{\sqrt{2\pi}}\Big(p_0^i + \frac{1}{\sqrt{2}}\sum_{n\neq 0}(\alpha_n^i{\rm e}^{in\sigma} + \tilde{\alpha}_n^i{\rm e}^{-in\sigma})\Big), \\ c^+(\sigma) =& \frac{1}{\sqrt{2\pi}}\sum_n c_n{\rm e}^{in\sigma}, \\ -\frac{1}{\sqrt{2}}b_{++} =& \frac{1}{\sqrt{2\pi}}\sum_n b_n{\rm e}^{in\sigma}, \\ A_\sigma(\sigma) =& \frac{1}{\sqrt{2\pi}}\Big(q_0 + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{1}{n}(\beta_n{\rm e}^{in\sigma} + \tilde{\beta}_n{\rm e}^{-in\sigma})\Big), \\ E(\sigma) =& \frac{1}{\sqrt{2\pi}}\Big(\pi_0 + \frac{1}{\sqrt{2}}\sum_{n\neq 0}(\beta_n{\rm e}^{in\sigma} + \tilde{\beta}_n{\rm e}^{-in\sigma})\Big), \\ A_\tau(\sigma) =& \frac{1}{\sqrt{2\pi}}\Big(\Lambda_0 + \frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{1}{n}\Lambda_n{\rm e}^{in\sigma}\Big).\end{aligned} \qquad (11)$$

On these components, the commutation and anticommutation relations (8) imply,

$$\begin{aligned}[\phi_0^i, p_0^j] =& i\delta^{ij}, & \\ [\alpha_m^i, \alpha_n^j] =& m\delta_{m+n}\delta^{ij}, & m,n\in Z,\quad m,n\neq 0, \\ [\tilde{\alpha}_m^i, \tilde{\alpha}_n^j] =& m\delta_{m+n}\delta^{ij}, & m,n\in Z,\quad m,n\neq 0, \\ \{b_n, c_m\} =& \delta_{n+m}, & m,n\in Z, \\ [q_0, \pi_0] =& i, & \\ [\beta_m, \beta_n] =& m\delta_{m+n}, & m,n\in Z,\quad m,n\neq 0, \\ [\tilde{\beta}_m, \tilde{\beta}_n] =& m\delta_{m+n}, & m,n\in Z,\quad m,n\neq 0,\end{aligned} \qquad (12)$$

while all other possible relations vanish.

Once we have quantized the theory in this way let us discuss the content of the Fock space made out of the operator components in (11). We interpret the α_n^i, $\tilde{\alpha}_n^i$, β_n and $\tilde{\beta}_n$ as harmonic oscillator raising and lowering operators for negative or positive n, respectively. In this interpretation ${\alpha_n^i}^\dagger = \alpha_{-n}^i$, $\tilde\alpha_n^{i\dagger} = \tilde\alpha_{-n}^i$, $\beta_n^\dagger = \beta_{-n}$, $\tilde\beta_n^\dagger = \tilde\beta_{-n}$, for $n > 0$, and so the Fock vacuum $|0>$ is the state which satisfies,

$$\alpha_n^i|0>=\tilde\alpha_n^i|0>=\beta_n|0>=\tilde\beta_n|0>=\beta_0|0>=b_n|0>=c_n|0>=b_0|0>=0,\quad n>0, \tag{13}$$

with $\beta_0 = \frac{1}{\sqrt2}(iq_0 - \pi_0)$. The Fock space is made out of the operators in (11) with $n < 0$, and by $\beta_0^\dagger$ and c_0, acting on $|0>$. This choice of vacuum allows to use the ordinary normal ordering prescription [4]. Notice that we have,

$$-\partial_\sigma\phi^i + \pi_\phi^i = \frac{1}{\sqrt{2\pi}}(p_0^i + \sqrt2\sum_{n\neq0}\alpha_n^i \mathrm{e}^{-in\sigma}), \tag{14}$$

and so we can identify α_n^i with the harmonic oscillator operators corresponding to the right-moving part of ϕ^i. A similar calculation for $\partial_\sigma\phi^i + \pi_\phi^i$ leads to interpret the operators $\tilde\alpha_n^i$ as the corresponding to the left-moving sector.

The computation of the BRST charge Q associated to the BRST current (7) is straightforward. First, notice that using (9) one obtains,

$$D_+\phi^i = \frac{1}{\sqrt2}(\partial_\sigma\phi^i + \pi_\phi^i). \tag{15}$$

Utilizing this fact and (7) together with the expansions (11) one finds for the BRST charge,

$$\begin{aligned} Q &= \frac{\sqrt{2\pi}}{2}\int_0^{2\pi} d\sigma\, J_+^B \\ &= \frac12\sum_{m,n} :(c_{-n}\tilde\alpha_{m+n}^i\tilde\alpha_{-m}^i - (n-m)c_{-n}c_{-m}b_{n+m}): \\ &\quad + 2i\sqrt\pi\alpha\omega^i\sum_n nc_{-n}\alpha_n^i - \rho c_0, \end{aligned} \tag{16}$$

where ρ is a c-number which accounts for the normal ordering ambiguities in Q and we have defined,

$$\tilde\alpha_0^i = \frac{1}{\sqrt2}p_0^i. \tag{17}$$

The form of Q is identical to the one found in the free case in [4]. As it was shown there, the operator Q is nilpotent only if

$$\alpha = -\frac{1}{\sqrt{2\pi}}, \qquad \rho = 0. \tag{18}$$

Furthermore, we may apply the BRST analysis of the Fock space carried out in [4] to our case. As a result, the states $|\Phi>$ of our Fock space which satisfy the BRST condition,

$$Q|\Phi>=0, \tag{19}$$

are those made only of the harmonic oscillator operators α_n^i and the ones corresponding to the vector field. This fact permits to define the following effective Hamiltonian in this reduced Fock space:

$$\begin{aligned} H^{\rm eff} = \int_0^{2\pi} d\sigma\,\Big(&\frac14(\partial_\sigma\phi - \pi_\phi^i)^2 - e^i(A_\sigma + A_\tau)(\partial_\sigma\phi^i - \pi_\phi^i) + \frac12 E^2 + E\partial_\sigma A_\tau \\ &+ \frac12 e^2((1-a)A_\tau^2 + 2A_\tau A_\sigma + (1+a)A_\sigma^2)\Big), \end{aligned} \tag{20}$$

i.e., $H^{\rm eff}$ is the form of H when restricted to act on the reduced Fock space defined by condition (19). This is because the coefficient of c_0 in the expression for Q in (16) is proportional to difference $H - H^{\rm eff}$. This reduced Fock space constitutes the physical Fock space of the theory. Notice that all the left-moving modes of ϕ^i as well as all the ghost components have dropped from H. In $H^{\rm eff}$ the dependence on the zero mode p_0^i is irrelevant since one of the consequences of condition (19) is that on the physical Fock space $p_0^i = 0$ [4]. Our next task is to obtain the spectrum of the theory in the physical Fock space. This will be achieved by diagonalizing the effective Hamiltonian $H^{\rm eff}$ in (20). In order to do that let us first of all determine A_τ from its equation of motion $\frac{\delta H}{\delta A_\tau}=0$. For $a\neq 1$ one has:

$$A_\tau=\frac{1}{e^2(a-1)}\big(e^i(\pi^i_\phi-\partial_\sigma\phi^i)+e^2A_\sigma-\partial_\sigma E\big). \tag{21}$$

Taking Fourier components in (21) we obtain :

$$\begin{aligned}\Lambda_0=&\frac{1}{e^2(a-1)}(e^ip^i_0+e^2q_0),\\ \Lambda_n=&\frac{n}{e^2(1-a)}\big(2ie^i\alpha^i_n+(n-\frac{e^2}{n})\beta_n+(n+\frac{e^2}{n})\tilde\beta^\dagger_n\big).\end{aligned} \tag{22}$$

From (22) and the commutation relations given in (12), it is simple to check that:

$$[\Lambda_n,\Lambda_m]=0,\quad m,n\neq 0. \tag{23}$$

In terms of the harmonic oscillator operators, the Hamiltonian $H^{\rm eff}$ can be written as a sum of two contributions corresponding to zero and non-zero modes :

$$H^{\rm eff}=H_z+\sum_{n>0}H_{nz}(n), \tag{24}$$

where,

$$H_z=\frac{\pi_0^2}{2}+\frac{e^2a^2}{2(a-1)}q_0^2, \tag{25}$$

and,

$$\begin{aligned}H_{nz}(n)=&\frac12{\alpha^i_n}^\dagger\alpha^i_n+\frac{i}{n}e^i\alpha^i_n(\tilde\beta_n-\beta^\dagger_n)+\frac14\big(1+\frac{e^2(1+a)}{n^2}\big)(\beta^\dagger_n\beta_n+\tilde\beta^\dagger_n\tilde\beta_n)\\ &\frac14(1-\frac{e^2(1+a)}{n^2})\beta_n\tilde\beta_n+\frac{e^2(a-1)}{4n^2}\Lambda^\dagger_n\Lambda_n+\text{h.c.}\end{aligned} \tag{26}$$

In deriving (25) we have used the fact that on the physical Fock space $p^i_0=0$. In (26) Λ_n must be understood as the dependent expression given in (22). Let

us consider first the zero-mode Hamiltonian. The terms in H_z correspond to a harmonic oscillator of frequency M , with

$$M^2=e^2\frac{a^2}{a-1}. \tag{27}$$

Notice that M^2 is positive (*i.e.*, there are no tachyons in the spectrum) when $a>1$. Henceforth we shall restrict ourselves to this range of the parameter a. On the other hand if we define the operator,

$$d=\frac{1}{\sqrt{2M}}(iMq_0-\pi_0), \tag{28}$$

which satisfies the commutation relation:

$$[d,d^\dagger]=1. \tag{29}$$

H_z can be written as:

$$H_z=Md^\dagger d. \tag{30}$$

In order to diagonalize $H_{nz}(n)$ let us define the operators:

$$\begin{aligned}d_n=&\frac{1}{M\sqrt{E_n}}\big(\frac{ia}{a-1}e^i{\alpha^i_n}^\dagger-\frac12(\Delta_++E_n)\tilde\beta_n-\frac12(\Delta_-+E_n)\beta^\dagger_n\big),\\ d_{-n}=&\frac{1}{M\sqrt{E_n}}\big(\frac{ia}{a-1}e^i\alpha^i_n+\frac12(\Delta_--E_n)\beta_n+\frac12(\Delta_+-E_n)\tilde\beta^\dagger_n\big),\\ a^1_n=&\frac{i}{e\sqrt n}e^i\alpha^i_n+\frac{\sqrt n}{ea}(\beta_n+\tilde\beta^\dagger_n),\\ a^2_n=&\frac{i}{e\sqrt n}(e^1\alpha^2_n-e^2\alpha^1_n),\end{aligned} \tag{31}$$

where $n>0$ and:

$$E_n=\sqrt{n^2+M^2},\quad \Delta_\pm=\frac{n^2\pm e^2a^2}{n(a-1)}. \tag{32}$$

Notice that in (31) $d^\dagger_n\neq d_{-n}$. It can be checked that these operators satisfy the following commutation relations:

$$\begin{aligned} [a_n^i, {a_m^j}^\dagger] &= \delta_{mn}\delta^{ij}, & n, m > 0, \\ [d_n, d_m^\dagger] &= \delta_{nm}, & n, m \neq 0. \end{aligned} \tag{33}$$

while any other commutators vanish. The fact that the definitions (31) lead to relations (33) imply that the transformation involved is unitary. In terms of these new harmonic oscillator operators the Hamiltonian $H_{nz}(n)$ can be written as:

$$H_{nz}(n) = n{a_n^i}^\dagger a_n^i + E_n(d_n^\dagger d_n + d_{-n}^\dagger d_{-n}). \tag{34}$$

The $n \neq 0$ spectrum matches perfectly well the zero-mode energy levels and in fact introducing (34) and (30) into (24) we have for the total Hamiltonian :

$$H^{\text{eff}} = \sum_{n>0} n{a_n^i}^\dagger a_n^i + \sum_{n\in Z} \sqrt{n^2 + M^2} d_n^\dagger d_n. \tag{35}$$

where we have denoted d by $d_{n=0}$. We thus see that the spectrum of the theory consists of a massive boson of mass M together with two massless right-handed excitations. Observe that only half ($n > 0$) of the modes appear in the massless contribution to H^{eff}, so these particles are really chiral. The spectrum is perfectly relativistic and in fact is identical to the two flavour chiral Schwinger model in which two right-handed massless fermions are coupled to the gauge field [15].

The transformation of the harmonic oscillator operators that we have used to diagonalize the Hamiltonian is nothing but a Bogoliubov transformation. This means that the particles that appear in the spectrum can be considered as collective modes of the oscillators that we used to build our reduced Fock space. The true vacuum of the theory is not the Fock vacuum that we defined in (13) but instead is a coherent superposition of our initial Fock states obtained by acting on $|0>$ with the Bogoliubov transformation.

It is important to point out that within this Siegel approach it is only possible to formulate the model with an even number of chiral bosons. This is so due to the no-ghost theorem proved in [4]. Otherwise one would not be able to obtain the physical states all with the same chirality. In general one would need to introduce an auxiliary field λ^{++} for each pair of chiral bosons. The generalization of our analysis to this general case is straightforward and one can easily check that one obtains the same spectrum that in the corresponding chiral Schwinger model.

An alternative approach to describe chiral bosons coupled to abelian gauge fields consists of the study of the Lagrangian of Floreanini and Jackiw [2] coupled to an $U(1)$ vector field. In this case it seems plausible that one may be able to show that a system with a single chiral boson is equivalent to the correponding Schwinger model.

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