The Nature of Deconfinement

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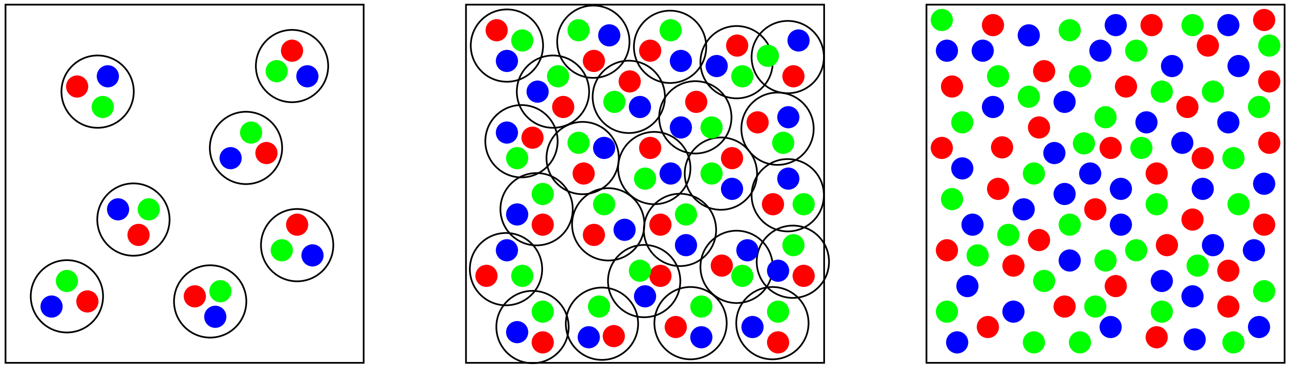
Santiago de Compostela **February 10, 2006**

What happens to strongly interacting matter when the density of constituents is increased?



Nucleon Gas   Nuclear Matter   Quark Matter

When many hadrons overlap, quarks cannot identify "their hadron", the concepts of a hadron and of confinement become meaningless, high quark density and colour screening forbid hadronic scales, there is a transition to a different state of matter.

1. The Problem

QCD predicts that with increasing temperature, hadronic matter undergoes a transition to a plasma of deconfined quarks and gluons

What kind of transition?

Thermodynamic critical behaviour, phase transition:

spontaneous symmetry breaking $\Rightarrow$ singular behaviour of partition function, discontinuous or divergent thermal observables

Two limits in QCD lead to thermodynamic critical behaviour:

- $m_q \to \infty$ **: pure gauge theory (glueball/gluon medium)**

  global Z_N symmetry breaking $\to$ deconfinement

- $m_q \to 0$ **: chiral limit (constituent/current quarks)**

  chiral symmetry restoration $\to$ effective quark mass shift

Order Parameters

deconfinement: Polyakov loop $L(T) \sim \exp\{-F_{Q\bar{Q}}/T\}$

$F_{Q\bar{Q}}$**: free energy of static** $Q\bar{Q}$ **pair for** $r \to \infty$

$$L(T) \begin{cases} = 0 & T < T_L \ \text{confinement} \\ \\ \neq 0 & T > T_L \ \text{deconfinement} \end{cases}$$

defines deconfinement temperature T_L

chiral symmetry restoration: chiral condensate $\chi(T) \equiv \langle \bar{\psi}\psi \rangle \sim M_q$

measures dynamically generated ('constituent') quark mass

$$\chi(T) \begin{cases} \neq 0 & T < T_\chi \ \text{chiral symmetry broken} \\ \\ = 0 & T > T_\chi \ \text{chiral symmetry restored} \end{cases}$$

defines chiral symmetry temperature T_χ

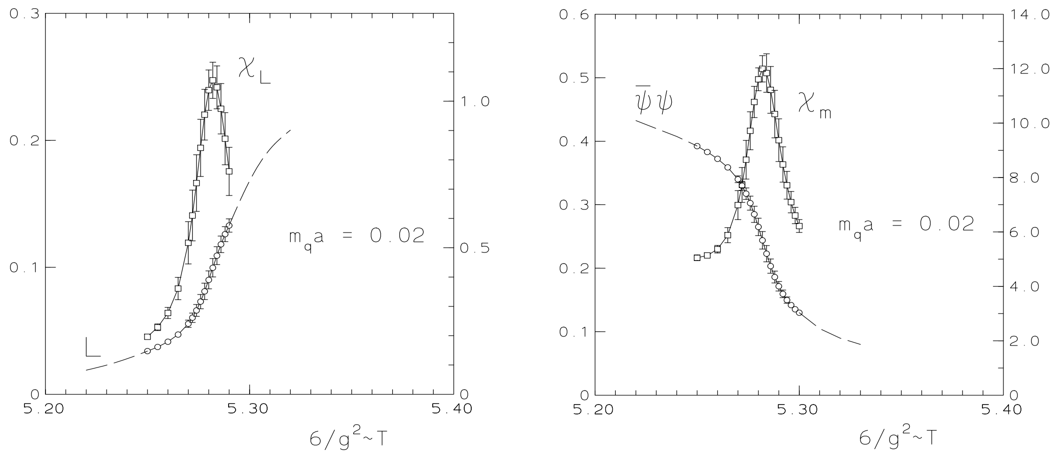
Caveat:

order parameters, associated T_x exist strictly only in limits $m_q \to 0, \infty$

But finite temperature lattice QCD finds:

$L(T)$, $\chi(T)$, energy density vary rapidly in small T interval for all m_q, define T_L and T_χ for all m_q

Relation T_L vs. T_χ ?

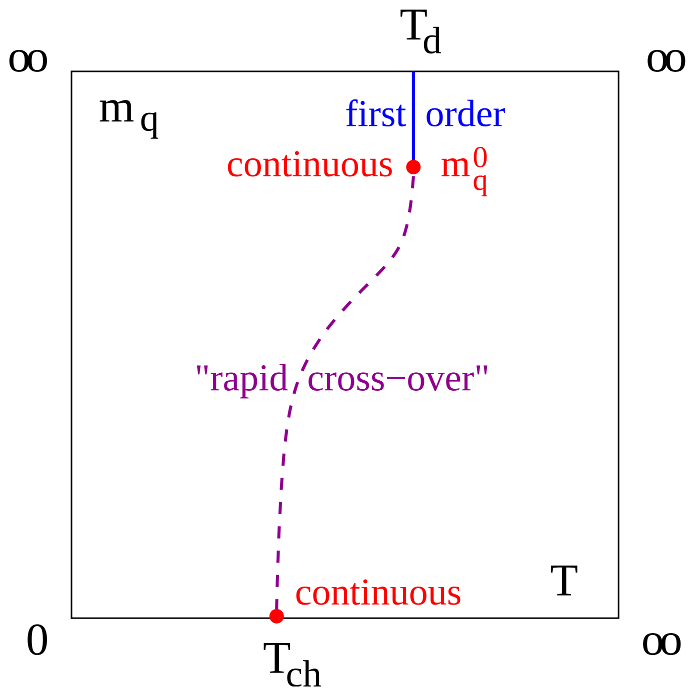


Polyakov loop and chiral condensate vs. temperature

- **Deconfinement, chiral symmetry restoration transitions coincide**
- **What transition structure for different values of m_q?**

Illustration: **QCD for colour SU(3), $N_f = 2$, $\mu_B = 0$**

- $\infty \leq m_q < m_q^0$
  first order (deconfinement)
- $m_q = m_q^0$
  second order (deconfinement)
- $0 < m_q < m_q^0$
  "rapid cross-over"
- $m_q = 0$
  second order (chiral)



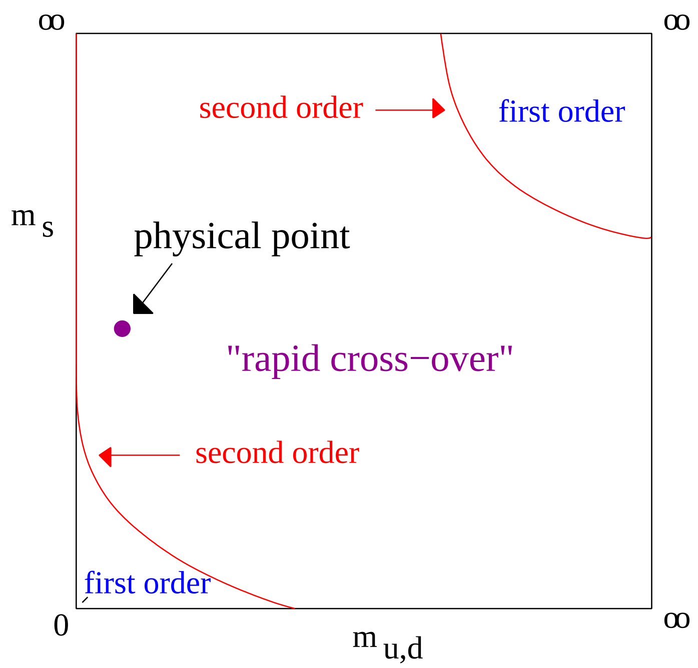
"rapid cross-over":

– **no spontaneous symmetry breaking,**

– **no singular behaviour of thermodynamic observables,**

– **but always rapid variation in narrow temperature interval**

Real world: **QCD for SU(3), $N_f = 3$, $m_q \neq 0$, $\mu_B = 0$**

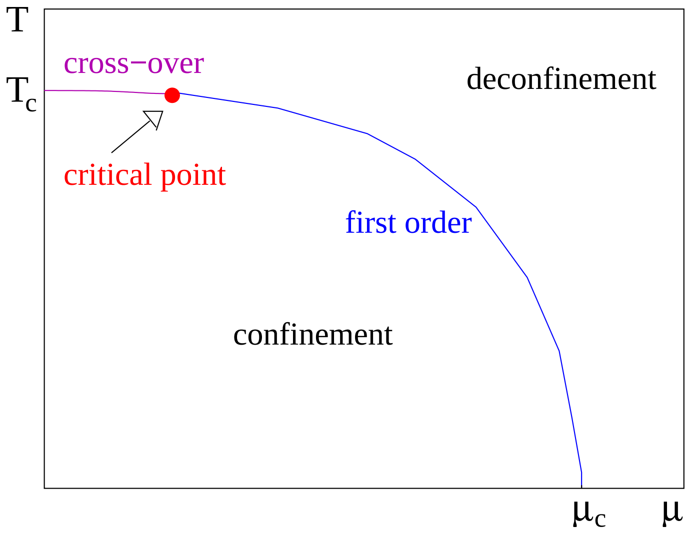


physical point of two light, one heavier quark: rapid cross-over

Finite baryon density, $\mu_B \neq 0$:

computer simulation difficulties, need new algorithms

First results:



⇒ **Deconfinement in the early universe, in high energy heavy ion collisions occurs as rapid cross-over.**

But what is a rapid cross-over?

How do the separated states of matter differ?

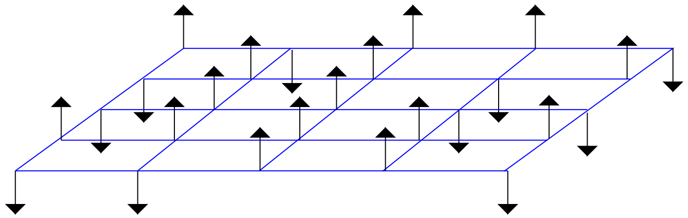
Thermodynamic critical behaviour, phase transitions well-defined: spontaneous symmetry breaking → non-analytic partition function

Is there a more general formulation of critical behaviour?

2. Critical Behaviour and Cluster Formation

Look for help in theory of spin systems, Ising model:

d**-dimensional lattice grid,** N^d **sites with spins** $s_i = \pm 1 \ \forall i = 1, ..., N^d$**,**
uniform next neighbor interaction $\mathcal{H} = -J s_i s_{i+1}$



partition function

$$Z(T, H\!=\!0) = \prod_{i=1}^{N^d} \sum_{s_i=\pm 1} \exp\{\, \beta J \sum_{i,j}^{nn} s_i s_j - \beta H \sum_i s_i\}$$

temperature $T = \beta^{-1}$**, external field** H**; take** $H = 0$

Partition function has global Z_2 symmetry:

$$s_i \to -s_i \quad \forall\ i = 1, ..., N^d$$

leaves sum over all states $Z(T, H=0)$ invariant

for high temperatures ($T \geq T_c$), state of system also invariant:

$\exists$ disorder, as many spins $\uparrow$ as $\downarrow$

for low temperatures ($T \leq T_c$), state of system not invariant:

$\exists$ order, more $\uparrow$ or more $\downarrow$, spontaneous symmetry breaking

State of system specified by order parameter

$$m(T, H = 0) = \frac{1}{Z(T, H = 0)} \prod_{i=1}^{N^d} \sum_i \left[\frac{\sum_i s_i}{N^d} \right] \exp\{ \beta J \sum_{i,j}^{nn} s_i s_j \}$$

not invariant under spin flip $s_i \to -s_i\ \forall\ i$:

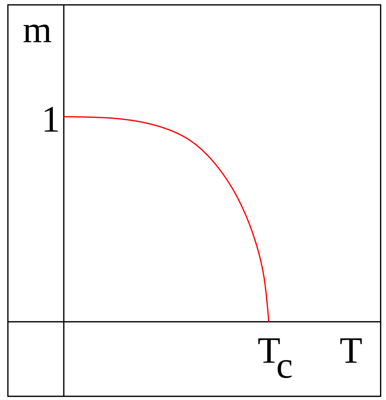
$$m(T, H = 0) \to -m(T, H = 0)$$

so that

$$m(T, H=0) \begin{cases} \neq 0 & \text{for ordered state, broken symmetry} \\ \\ = 0 & \text{for disordered state, symmetry} \end{cases}$$

In thermodynamic limit $N \to \infty$:

$m(T, H=0)$ **is not analytic**



$$m(T) \sim \begin{cases} (T_c - T)^\beta > 0 & \forall\, T < T_c \\ \\ 0 & \forall\, T > T_c \end{cases}$$

$\Rightarrow$ **critical exponent** β

Other observables $\to$ other critical exponents, scaling, renormalization group theory, universality classes, ...

Alternative view of Ising critical behaviour (consider $d = 2$)

domain formation & fusion

With decreasing temperature,

$\exists$ larger and larger clusters of like-sign spins, but for $T \geq T_c$: $N_\uparrow = N_\downarrow$

Below T_c,

$N_\uparrow > N_\downarrow$, infinite cluster of $\uparrow$, finite cluster of $\downarrow$ (or vice versa)

<u>Divergence of cluster size = Percolation</u>

Percolation:

geometric critical behaviour, critical exp'ts, universality classes, ...

$\Rightarrow$ Is the geometric critical behaviour,

defined as percolation of spin clusters,

equivalent to the thermal critical behaviour,

defined as onset of spontaneous magnetization? $\Leftarrow$

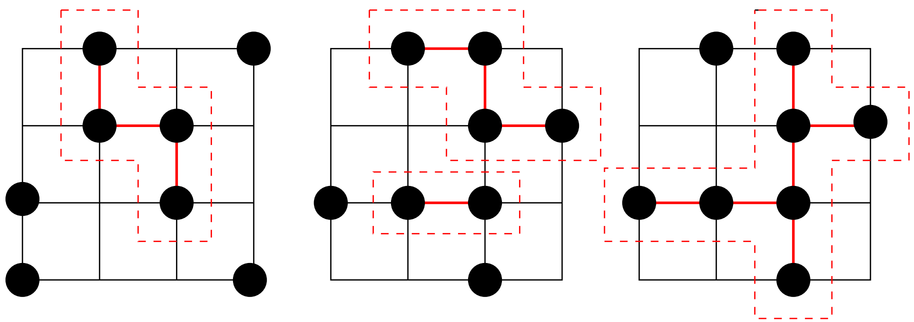
NB: "equivalent" = same T_c, same critical exponents

(same universality class)

Answer: YES, if... **Fortuin & Kasteleyn, Coniglio & Klein**

if... clusters are defined the right way:

- **set of adjacent like-sign spins;**
  this alone not enough, random like-sign combinations must be excluded
- **bonds between like-sign spins with** $p = 1 - \exp[-2J/kT]$
- **cluster** $\equiv$ **set of bonded, adjacent, like-sign spins**



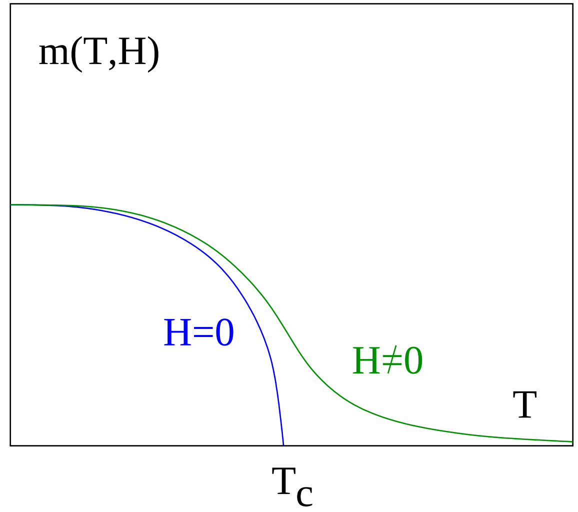
Ising model (any d**): cluster percolation = onset of magnetization**

identical critical behaviour

3. The Survival of Percolation

In the presence of an external magnetic field ($H \neq 0$), Z_2 is explicitly broken, the Ising partition function is analytic, there is no more thermal critical behaviour

For $H \neq 0$, $m(T,H) \neq 0 \; \forall \; T$

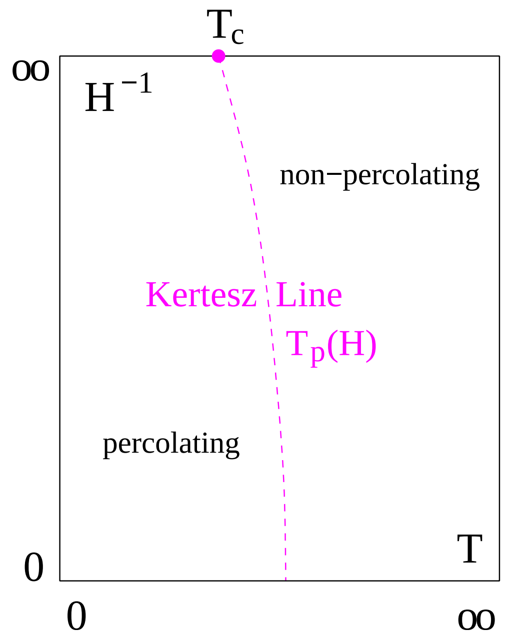


Percolation survives for all H, defines Kertész line $T_p(H)$ such that

$$T(H) \begin{cases} \leq T_p(H) & \exists \text{ cluster percolation} \\ \\ > T_p(H) & \text{no cluster percolation} \end{cases}$$

Ising model percolation structure

(Kertész line)



How to define Kertesz line $T_p(H)$?

- percolation temperature for (C-K) clusters at given H
- cluster size behaviour; number $n(s)$ for clusters of size s

J.-S. Wang

$$n(s) \sim s^{-\tau} \exp\{-hs - \Gamma(t)s^{\sigma}\}$$

with $h = H/kT$, $t = |T - T_c|$, $\sigma \simeq 2/3$

- **surface tension $\Gamma(t)$ constitutes order parameter for percolation transition when $H \neq 0$**

$$\Gamma(T) \sim \begin{cases} (T_p - T)^x > 0 & \forall\ T < T_p \\ \\ 0 & \forall\ T > T_p \end{cases}$$

in terms of physics in the percolating vs. non-percolating phase

Conclude:

geometric critical behaviour $\sim$ cluster percolation

– more general than thermal critical behaviour

– equivalent to thermal critical behaviour in specific limiting cases

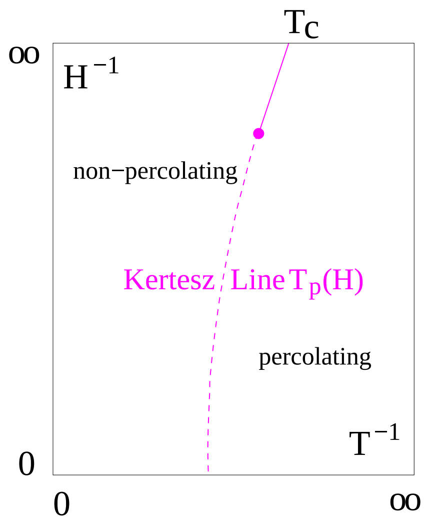
Compare phase structure of spin and gauge theories:

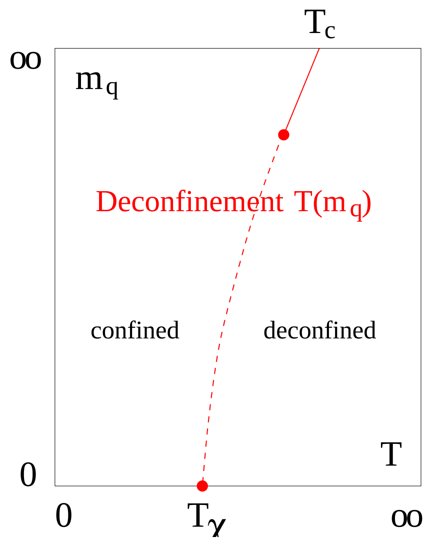
- **Ising (Z_2), $SU(2)$ in same universality class**
- **3-state Potts (Z_3), $SU(3)$ in same universality class**

QCD with colour SU(3), three equal mass flavours

SU(3) gauge theory, Z_3 spin theory for $H = 0$: 1^{st} order transition

(spin theory T^{-1}, H^{-1} $\sim$ gauge theory T, m_q)





Conjecture:

– **in general, deconfinement $\sim$ cluster percolation**

– **in the limit $m_q \to \infty$, cluster percolation $\equiv$ thermal transition**

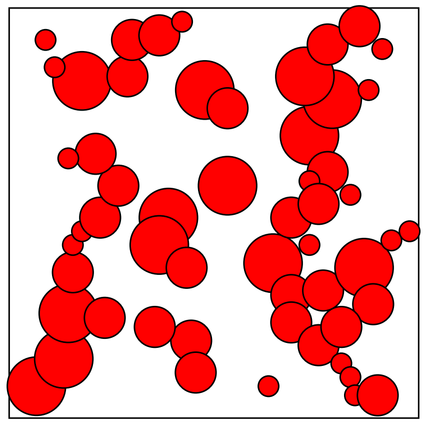
– **rapid cross-over for $m_q < m_q^0$ is reflection of cluster percolation**

4. Deconfinement and Hadron Percolation

Consider hadrons as spheres of radius $r_h \simeq 0.8$ fm

percolation occurs for density $$n_c = \frac{0.34}{(4\pi/3)\, r_h^3} \simeq 0.16 \text{ fm}^{-3}$$

$\Rightarrow$ **formation of hadronic matter**



for $n \leq n_c$: only isolated hadrons, clusters

for $n = n_c$: connected hadronic medium

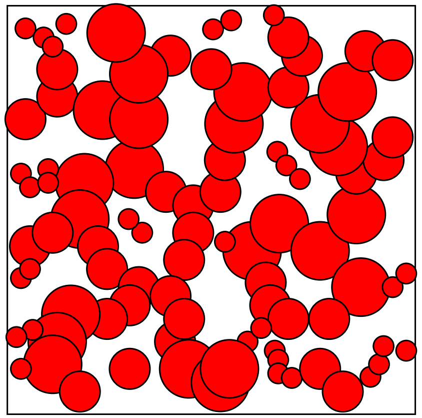
31 % hadronic clusters
69 % empty space

for $n \geq n_c$: both "media" percolate

When does the percolating vacuum disappear? Or, starting from high density side, when does vacuum first percolate?

Percolation condition for

"hadronic size" vacuum bubbles $\bar{n}_c = \frac{1.24}{(4\pi/3)\, r_h^3} \simeq 0.58 \text{ fm}^{-3}$



for $n = \bar{n}_c$**: end of connected vacuum**

69 % hadronic clusters

31 % empty space

for $n \geq \bar{n}_c$**: only isolated vacuum bubbles**

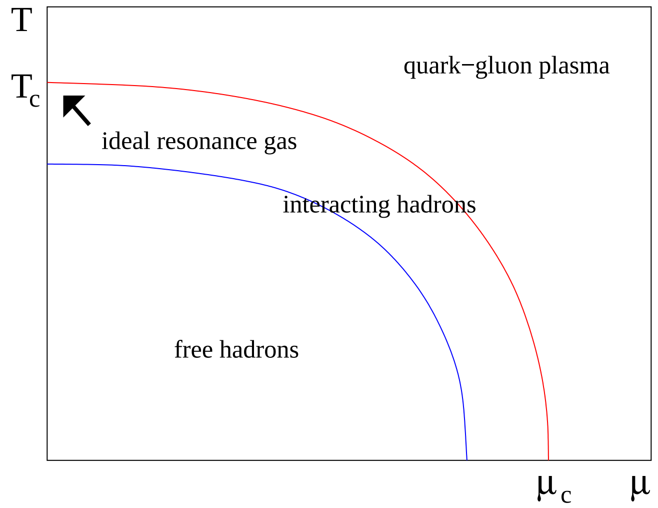
in dense interacting matter

Deconfinement as percolation:

when a hadronic medium becomes so dense that only isolated vacuum bubbles survive, then it becomes a quark-gluon plasma

Any predictive power?

confinement transition
⇒ **interacting medium of hadrons**



at $\mu \simeq 0$**:**
interaction is resonance-dominated
⇒ **ideal gas of hadrons and hadron resonances**

calculate resonance gas density $n(T)$

$n_h(T) = \bar{n}_c$ **determines critical temperature:**

$$T_c \simeq 170 \text{ MeV}$$

hadron size + percolation ⇒ deconfinement

Summary

1. **The Problem:**

   is there a general theory of deconfinement?
   what is a rapid cross-over?

2. **Critical Behaviour and Cluster Formation:**

   Ising Model: for $H = 0$, geometric = magnetic critical behaviour;
   geometric critical behaviour: cluster percolation

3. **The Survival of Percolation:**

   Ising Model: for $H \neq 0$, no more magnetic critical behaviour;
   percolation survives for all H, defines different states of matter;
   conjecture: deconfinement in general is cluster percolation

4. **Deconfinement and Hadron Percolation:**

   deconfinement as the disappearance of connected vaccuum;
   resonance gas $\rightarrow T_c$