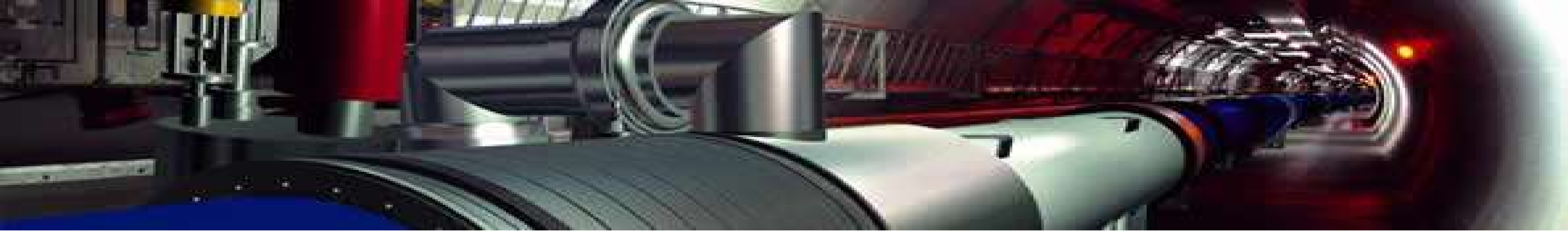
pQCD @ LHC

Germán Rodrigo
IFIC Valencia

Santiago de Compostela, Feb 2006

Outline

- Why higher orders at hadron colliders?
- Twistors, MHV amplitudes and recursion relations
- Conclusions



Particle physics has entered the LHC era

goal of the LHC:

proton-proton collisions
$\sqrt{s}$ = 14 TeV

→ **NEW PHYSICS** at the TeV energy scale

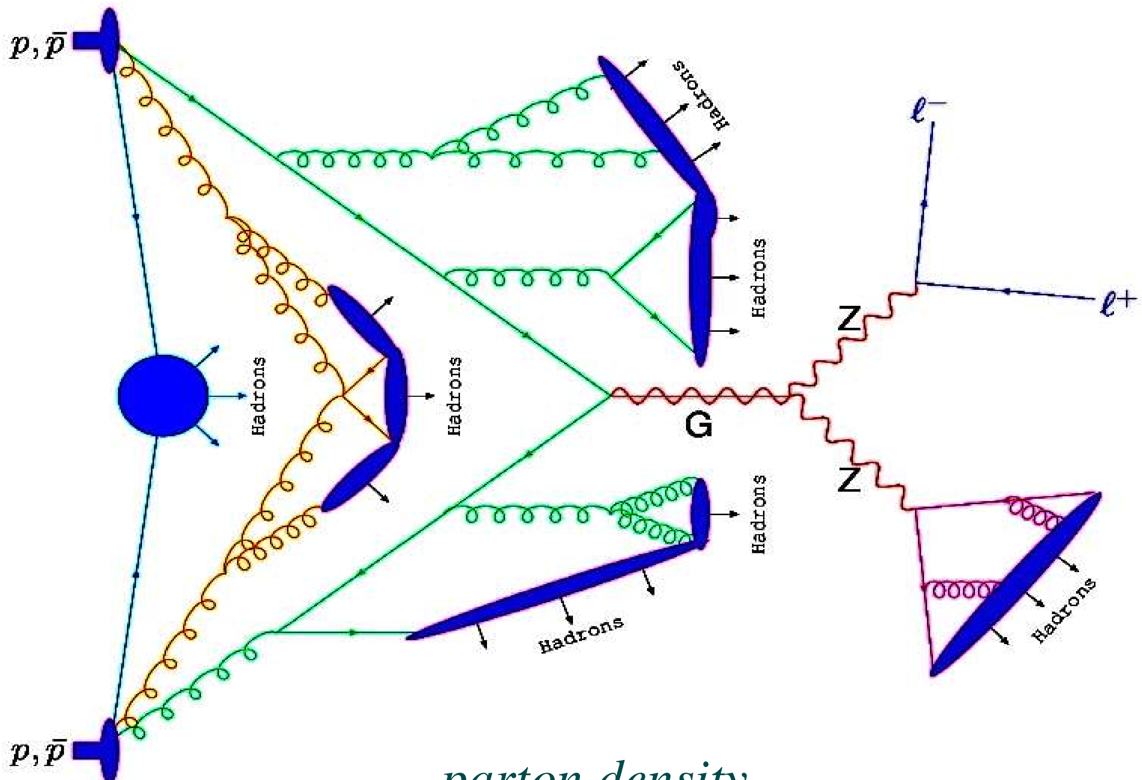
The LHC is a **QCD machine**

accurate QCD predictions for high-multiplicity final states

- to evaluate signal and background processes for new physics searches
- to explore QCD in a new high-energy regime

QCD is the toolkit for discovering physics beyond the SM

Hadronic colliders



FACTORIZATION:

- long distance (hadronic, M_{had})
- short-distance (partonic, $Q >> M_{had}$)

factorization violation is power supressed ~ $O(M_{had}/Q)^p$

parton density
PDF

partonic cross-section

$$\sigma(p_1, p_2) = \sum_{ab} \int dx_1 dx_2 f_1(x_1, \mu_F^2) f_2(x_2, \mu_F^2) \sigma_{ab}(x_1 p_1, x_2 p_2; \mu_F^2, \mu_R^2)$$

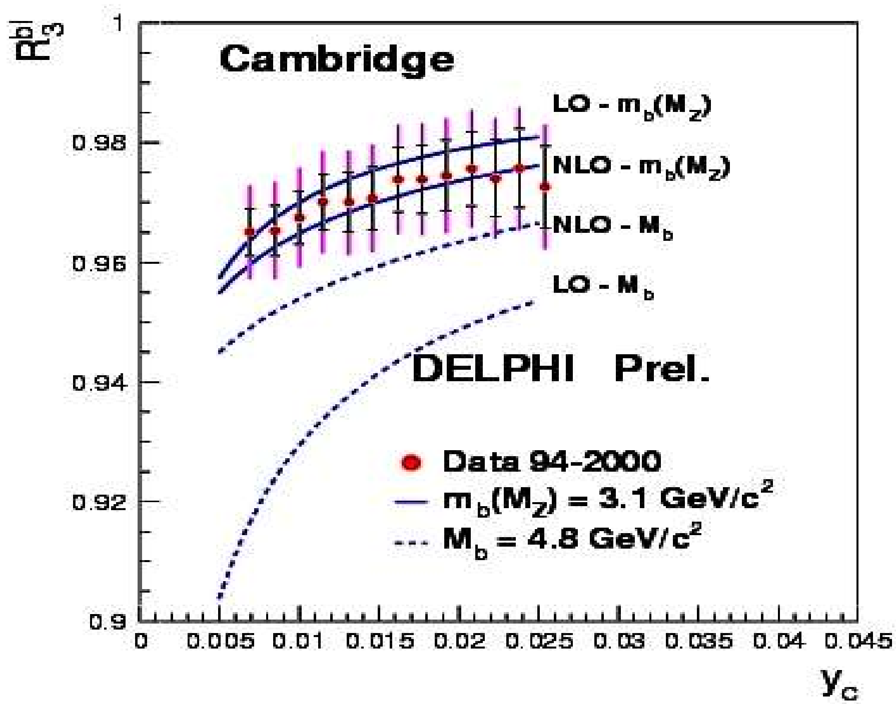
μ_F, μ_R: *factorization and renormalization scale*

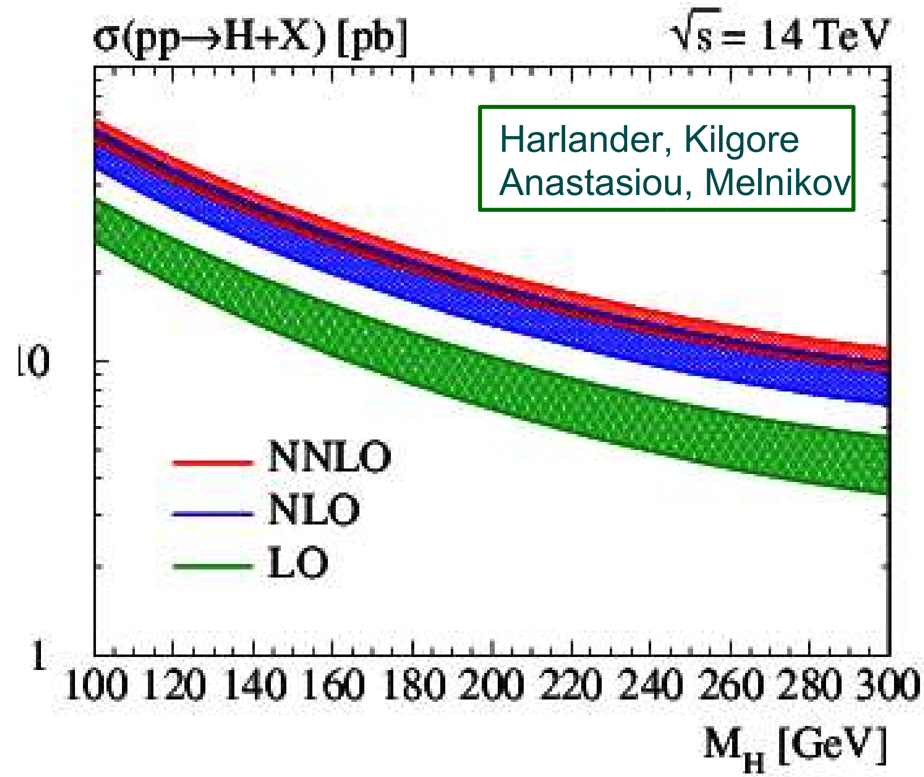
$\mu_F \sim \mu_R \sim Q$ *hard scale*

precision QCD at hadron colliders

what limits the precision?

- the order of the perturbative expansion
- the uncertainty in the input parton distribution functions





- ➤ Poor (off) description from LO predictions
- ➤ **NLO** first reliable estimate of the central value
- ➤ **NNLO** first serious estimate of the error

Anatomy of a NNLO calculation

2 parton final states

2-loop ⊗ tree-level

$|\text{1-loop}|^2$

3 parton final states

1-loop ⊗ tree-level

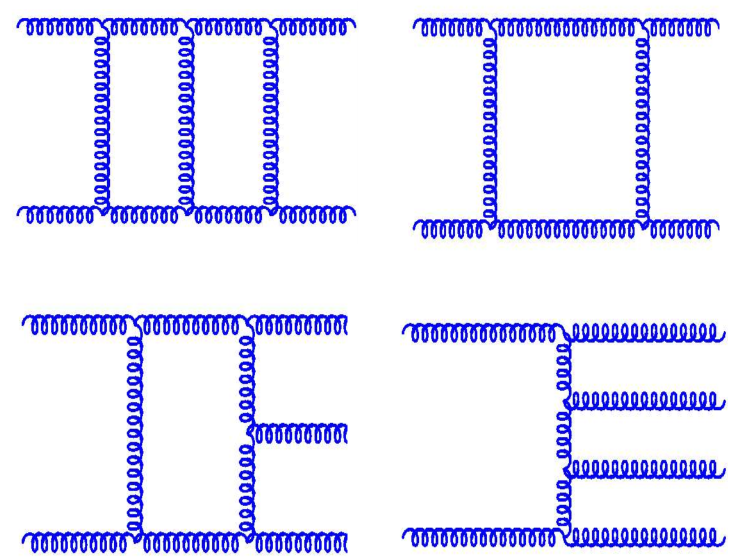
$|\text{tree-level}|^2$: $2+1_{\text{unresolved}}$

4 parton final states

$|\text{tree-level}|^2$: $2+2_{\text{unresolved}}$

$3+1_{\text{unresolved}}$

$p\,p \to jet + X$

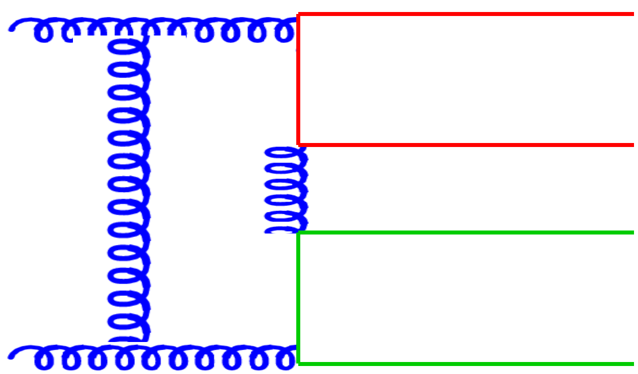


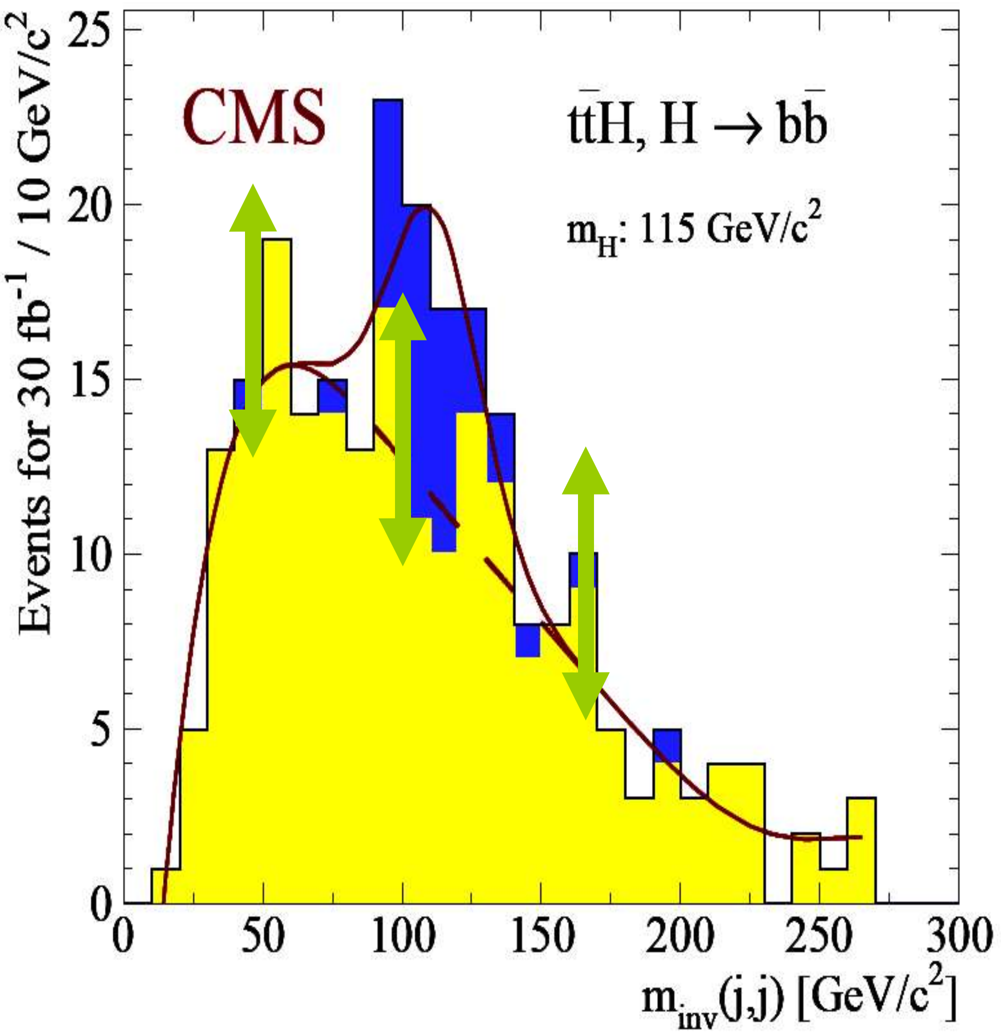
Collinear and soft singularities exactly cancel between tree-level and loop contributions

not all NLO corrections are known!

the more external coloured particles, the more difficult the NLO pQCD calculation

Example: $pp \rightarrow t\bar{t}b\bar{b} + X$ background to ttH

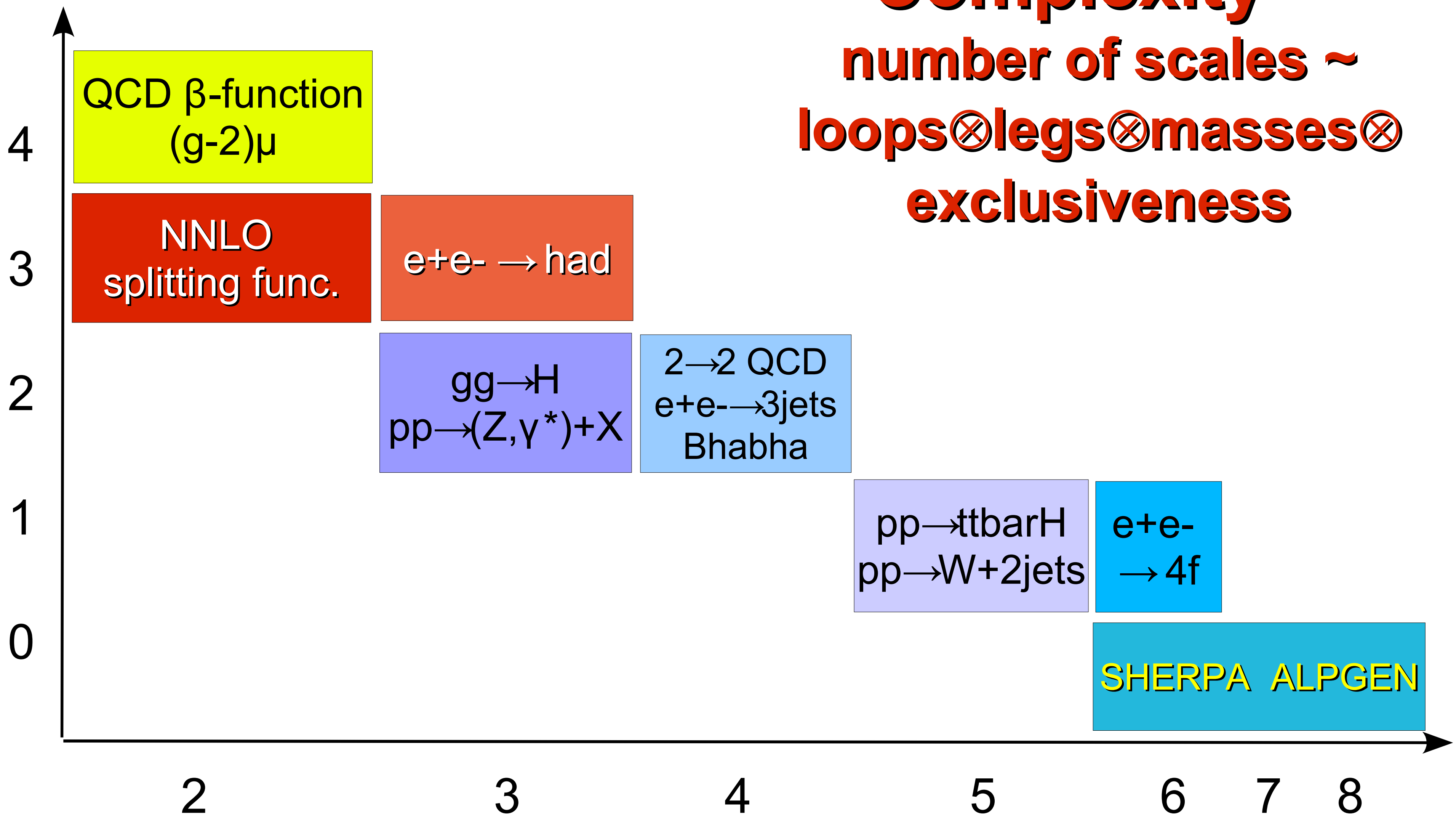




LO $O(\alpha_s^4)$ cross section has a large renormalisation scale dependence!

Loops and Legs

Complexity ~
number of scales ~
loops⊗legs⊗masses⊗
exclusiveness



NLO whish list

1.	$pp \to W\,W + jet$	
2.	$pp \to H + 2\; jets$	background to VBF Higgs production
3.	$pp \to t\,\bar{t}\,b\,\bar{b}$	
4.	$pp \to t\,\bar{t} + 2\; jets$	background to $t\,\bar{t}\,H$
5.	$pp \to W\,W\,b\,\bar{b}$	
6.	$pp \to V + 3\; jets$	general background to new physics
7.	$pp \to V\,V + 2\; jets$	background to $W\,W \to H \to WW$
8.	$pp \to V\,V\,V + jet$	background to SUSY trilepton

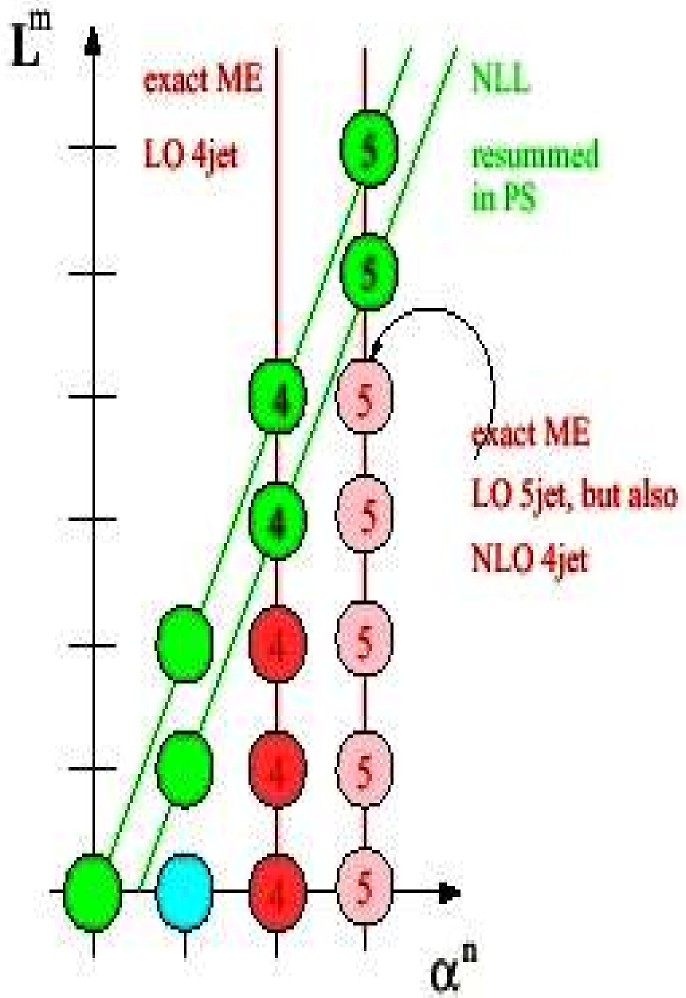
NLO whish list

1.	$pp \to W\,W + jet$	
2.	$pp \to H + 2\ jets$	background to VBF Higgs production
3.	$pp \to t\,\bar{t}\,b\,\bar{b}$	
4.	$pp \to t\,\bar{t} + 2\ jets$	background to $t\,\bar{t}\,H$
5.	$pp \to W\,W\,b\,\bar{b}$	
6.	$pp \to V + 3\ jets$	general background to new physics
7.	$pp \to V\,V + 2\ jets$	background to $W\,W \to H \to WW$
8.	$pp \to V\,V\,V + jet$	background to SUSY trilepton

automate

Resummations

pQCD at fixed order (LO,NLO,...) not always reliable



Hard processes are often multiscale, when the hard scales are very different large logarithms

$$L = \log \frac{Q_1^2}{Q_2^2}$$

in many cases double logs exponentiate

$$\sigma = \sigma^{(0)} \exp\left(L\, g_1(\alpha_S L) + g_2(\alpha_S L) + \alpha_S\, g_3(\alpha_S L) \right)$$

LL
leading log

NLL
next-to-leading

NNLL

NNLL only in few cases

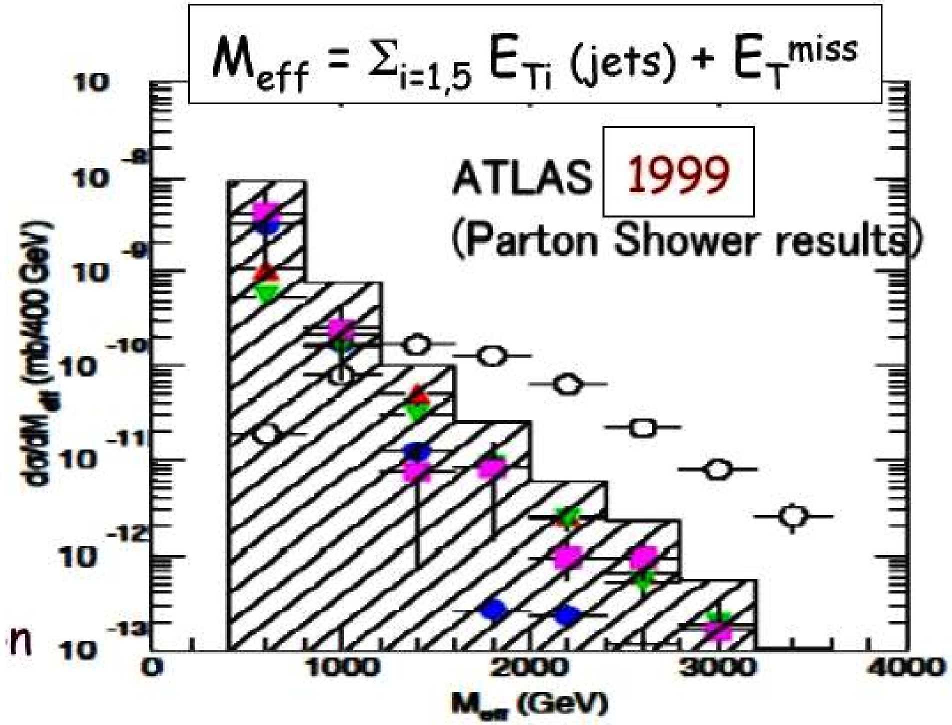
σ_{tot} and Q_T distribution for $g\,g \to H$

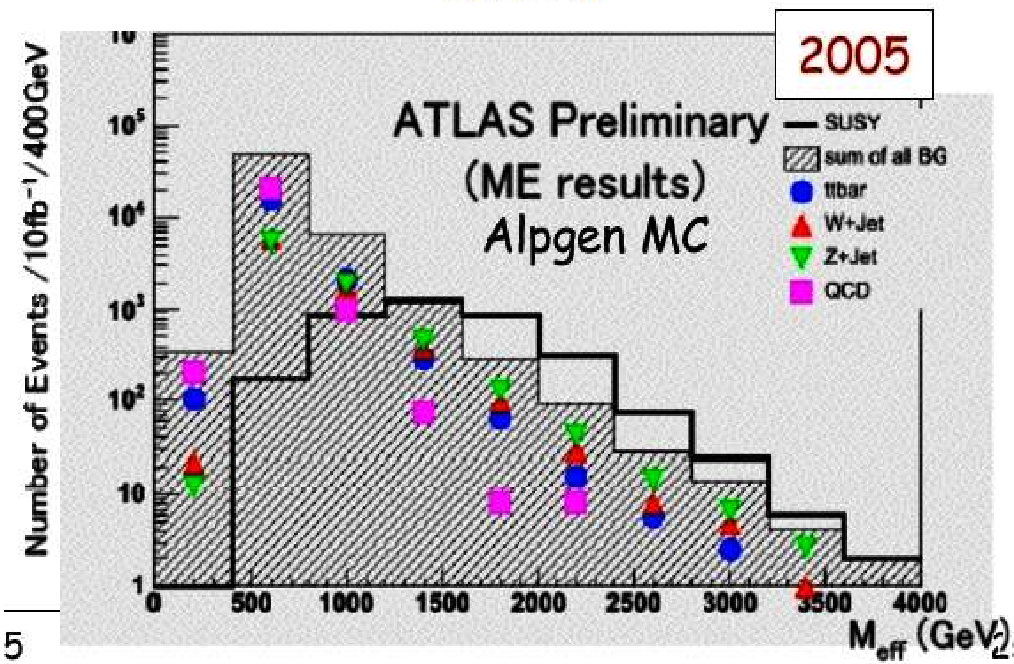
Monte Carlo tools

Parton shower
Monte Carlo underestimate the high-p_T region

SUSY signal less clear with **Matrix Element** (ME) Monte Carlo

adequate Monte Carlo tools needed to describe backgrounds





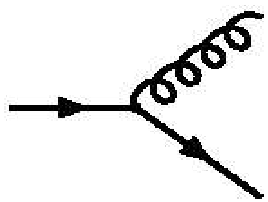
Perturbative asymmetries in nucleon's sea

[Catani, de Florian, GR, Vogelsang]

NNLO perturbative evolution generates a flavour asymmetry which is proportional to the valence content of the nucleon

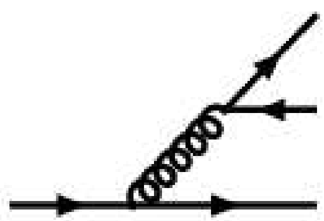
$$(s-\bar{s})(Q^2)=U^{(-)}(Q,Q_0)\left[(s-\bar{s})(Q_0^2)+\frac{1}{N_f}\left(\frac{U^{(V)}(Q,Q_0)}{U^{(-)}(Q,Q_0)}-1\right)\boldsymbol{f^{(V)}(Q_0^2)}\right]$$

at **LO**



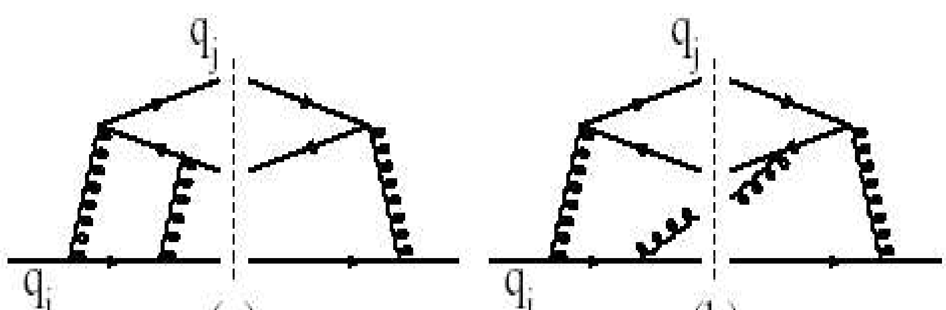
no $q \rightarrow q'$ $\quad P^S_{qq'}=0$

at **NLO**



$q \rightarrow q\,q'\,\bar{q}'$ but $\quad P^S_{qq'}=P^S_{q\bar{q}'}$

at **NNLO**

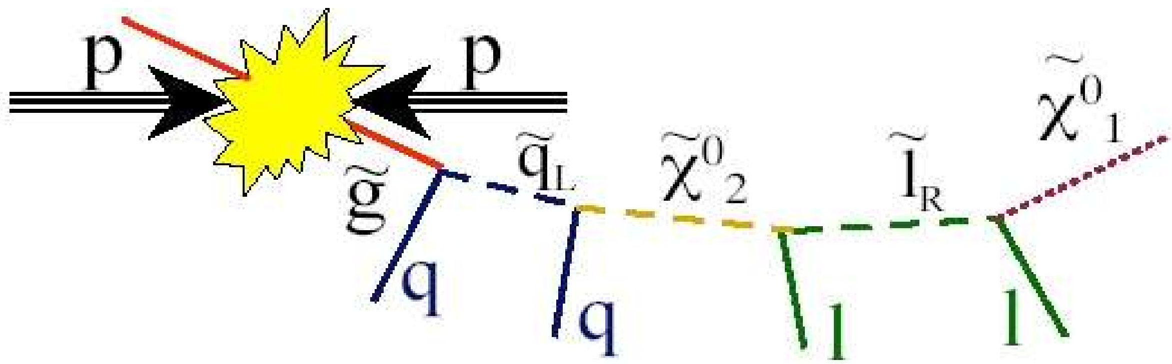


(a) (b)

$$P^S_{qq'} \neq P^S_{q\bar{q}'}$$

Multi-partonic amplitudes

Multi-jet production is the main signature at LHC, both for signal and background



Brute force calculation soon saturates computer capacity

- number of Feynman diagrams grows exponentially
- overlapping IR singularities at higher-orders
- complicated structure of multi-partonic phase-space

Key to efficient computation is recycling:

Der Grüne Punkt –

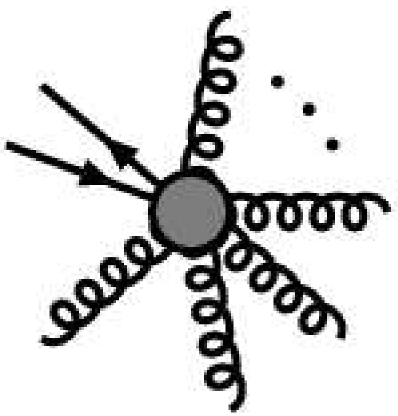
Duales System Deutschland AG

Multipartonic amplitudes

Witten's idea [hep-th/0312171] of a duality between **SYM** and topological string theories in twistor space

➡

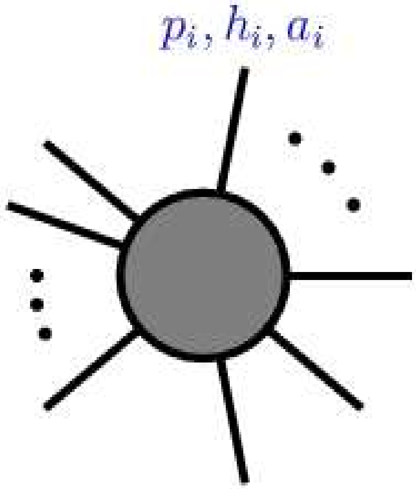
new developments for the calculation of multipartonic scattering amplitudes



Recursion relations

- **Off-shell** (Berends-Giele 1987)
- **MHV** (CSW Cachazo-Svrcek-Witten)
- **BCFW** (Britto-Cachazo-Feng-Witten)

Helicity basis+colour decomposition



Expressions simplify by using “right variables”

(1) Four-dimensional spinors of definite helicity

$$|p^{\pm}\rangle = \frac{1}{2}(1 \pm \gamma_5)\psi(p) \quad , \qquad \langle p^{\pm}| = \overline{\psi_{\pm}(p)}$$

$$\langle i\, j\rangle = \langle i^-|j^+\rangle, \qquad [i\, j] = \langle i^+|j^-\rangle, \qquad s_{ij} = \langle i\, j\rangle [j\, i]$$

Vector polarization
$$\epsilon_{\mu}^{\pm}(k,\xi) = \pm \frac{\langle \xi^{\mp}|\gamma_{\mu}|k^{\mp}\rangle}{\sqrt{2}\langle \xi^{\mp}|k^{\pm}\rangle}$$
equivalent to ***axial*** *gauges*

(2) for n-gluons, tree level

$$M_n(\{p_i, h_i, a_i\}) = \sum_{P(1,\dots,n)} Tr(T^{a_1} \dots T^{a_n})\, A_n(\{p_i, h_i\})$$

SU(N_C) generators in the fundamental representation

colour subamplitude: momenta and helicities

Colour ordering

n-gluon amplitude

n	# diagrams	# colour-ord diagrams
4	4	3
5	25	10
6	220	36
7	2485	133
8	34300	501
9	559405	1991
10	10525900	7225

Off-shell recursion relations

[Berends,Giele]

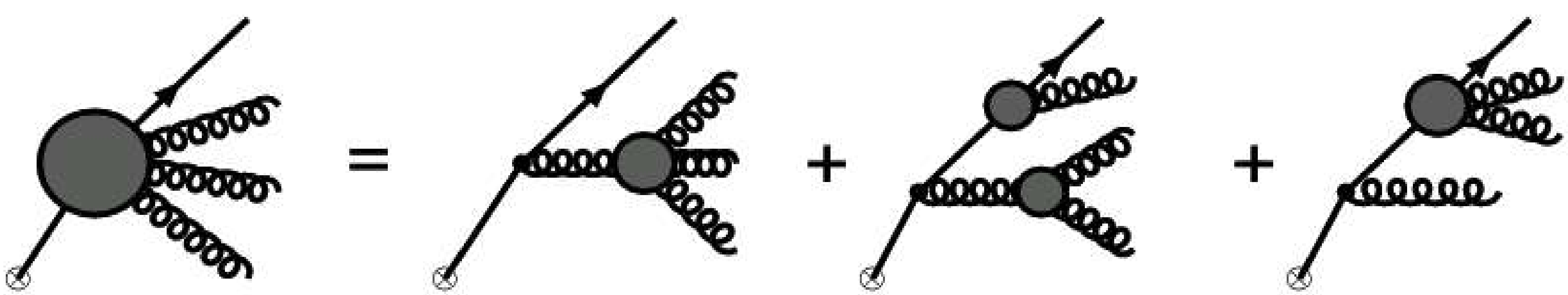
$$M(1_q;2,\ldots,n-1;n_{\bar{q}})=\sum_{P(2,\ldots,n-1)}(T^{a_2}\ldots T^{a_{n-1}})\,A(1_q;2,\ldots,n-1;n_{\bar{q}})$$

Off-shell spinorial currents

$$S(1_q;2,\ldots,m)=-\sum_{k-1}^{m-1}S(1_q;2,\ldots,k)\,J(k+1,\ldots,m)\frac{1}{p_{1,m}-m}$$

the gluonic current particularly simple for some helicity configurations

$$J^{\mu}(i^{+},\ldots,j^{+})=\frac{\langle\xi|\gamma^{\mu}p_{i,j}|\xi\rangle}{\sqrt{2}\langle\xi i\rangle\langle i(i+1)\rangle\ldots\langle j\xi\rangle}$$



MHV amplitudes

Multi-gluonic amplitudes at tree level

all gluon helicities positive
or only one negative helicity

$$A_n(1^+,\ldots,m^\pm,\ldots,n^+)=0$$

two negative helicities (***Maximal Helicity Violating*** amplitudes)
rather simple [Parke &Taylor, 1986]

$$A_n(1^+,\ldots,m_1^-,\ldots,m_2^-,\ldots,n^+)=i\frac{\langle m_1 m_2\rangle^4}{\langle 12\rangle\langle 23\rangle\ldots\langle (n-1)n\rangle\langle n1\rangle}$$

proved via ***recursion relations*** [Berends-Giele, Mangano-Parke-Xu,1988]

then next-to-**MHV**, and so on

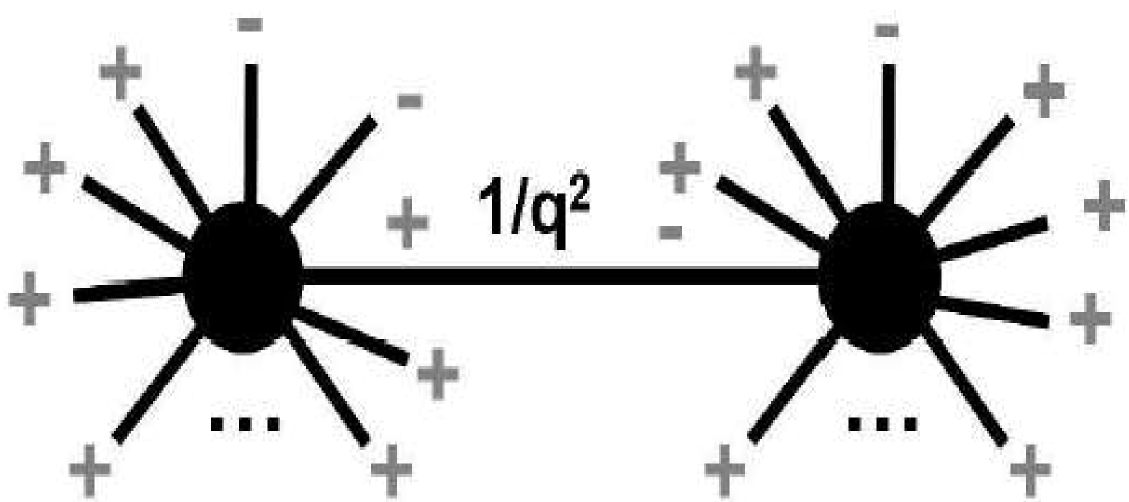
MHV vertices

Motivated by **twistor-space** structure [Cachazo, Svrcek and Witten (**CSW**)] define ***off-shell MHV*** vertices based on Parke-Taylor amplitudes

$$V(1^-,2^-,3^+,\ldots,n^+;P^+)=i\frac{\langle 1\,2\rangle^4}{\langle 12\rangle\ldots\langle (n-1)\,n\rangle\langle n\,P\rangle\langle P\,1\rangle}$$

continue spinor off-shell ($P^2\neq 0$): $\langle i\,P\rangle=\eta\sum_{j=1}^{n}\langle i^-|k_j|q^-\rangle$
where $P=k_1+k_2+\ldots+k_n$, and q auxiliary, $q^2=0$

Non-MHV amplitudes obtained by sewing MHV vertices through scalar propagators



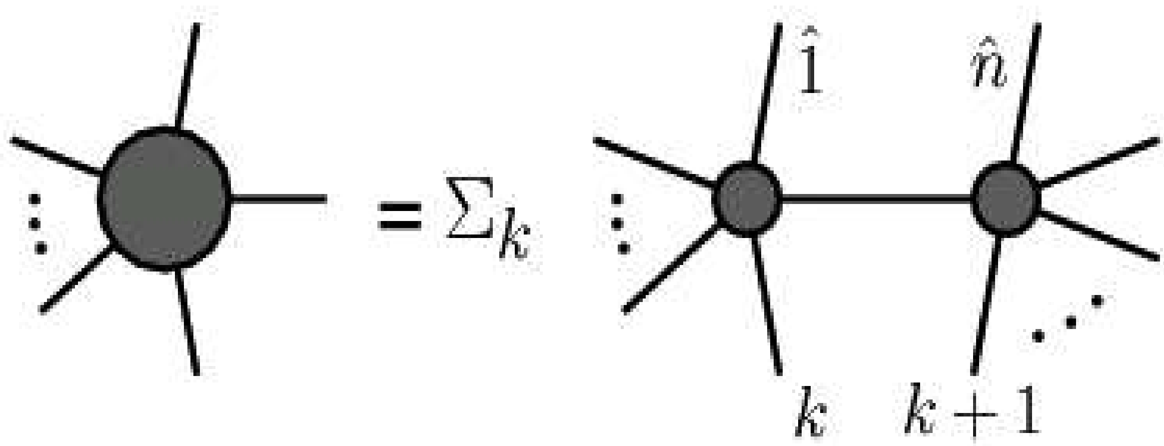
BCFW recursion

Reconstruct scattering amplitude from its singularities

add $z\eta^{\mu}$ on one external particle and subtract it on another such that the shift leaves them on-shell

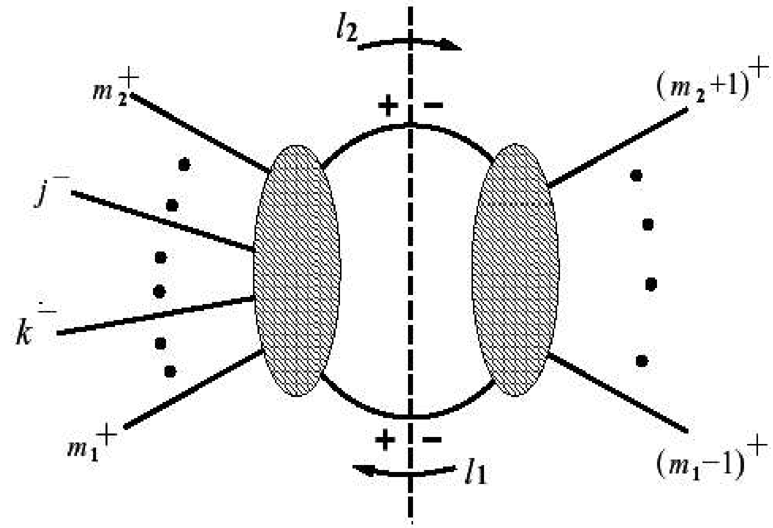
$$0 = \frac{1}{2\pi i}\oint_{C\,at\,\infty}\frac{A(z)}{z} = A(0) - \sum_{z_i}\frac{Res_{z_i}A(z)}{z_i}$$

has the correct residue at any multiparticle pole



$$A_n(1,2,\ldots,n) = \sum A_L(\hat{1},2,\ldots,-\hat{p}_{1,k})\frac{i}{p_{1,k}^2}A_R(\hat{p}_{1,k},k+1,\ldots,\hat{n})$$

Loops



- Loops using unitarity
  [Brandhuber,Spence,Travaglini,Bern, Dixon, Kosower, Bena, Roiban, Britto,Cachazo, Feng]

Most results apply to $\mathcal{N}$=4 ***supersymmetric Yang-Mills***

at tree level QCD is effectively supersymmetric

at one-loop:

$$\textbf{QCD}_{\textbf{gluons}} = (\mathcal{N}=4) + (\mathcal{N}=1) + \text{scalar}$$

pure QCD [Bern, Dixon, Kosower]

- Two-loop ($\mathcal{N}$=4) [Buchbinder, Cachazo][Anastasiou,Bern,Dixon,Kosower]

Massive particles

Twistors developed for massless theories

- Higgs in the heavy top limit [Badger, Dixon,Glover, Khoze]

  $H = \Phi + \Phi^\dagger$ (MHV-like vertices)

- EW bosons [Bern,Forde,Kosower,Mastrolia] external current
- BCFW for massive scalars, vector bosons and fermions

  [Badger,Glover,Khoze,Svrcek]

  explicit calculations with heavy scalars [Forde,Kosower]

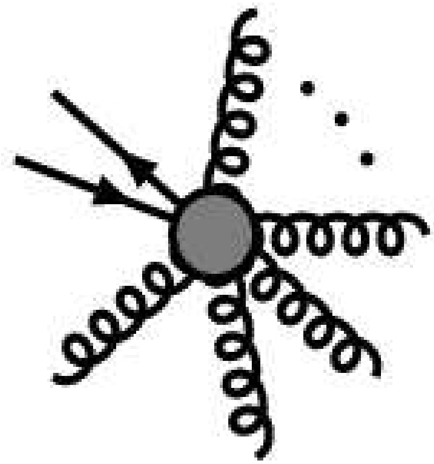
  and heavy fermions [GR] using Berends-Giele

LHC is a top factory

(NLO whish list)

Multipartonic amplitudes
for heavy scalars and fermions

[Ferrario, GR, Talavera]



Combining off-shell and BCFW recursion relations

$$A_n(1_s;2^+,\dots,n-1^+,n_s)=i\,m^2\frac{[2|\prod_{k=3}^{n-2}(y_{1,k}-\not{p}_k\,\not{p}_{1,k-1})|n-1]}{y_{12}\,y_{1,3}\cdots y_{1,n-2}\langle\langle 2,n-1\rangle\rangle}$$

where $\langle\langle i,j\rangle\rangle=\langle i(i+1)\rangle\dots\langle(j-1)\,j\rangle$ and $y_{1,k}=p_{1,k}^2-m^2$

SUSY-like Ward identities [Schwinn,Weinzierl]

$$A_n(1_q^+;2^+,\dots,n-1^+;n_{\bar{q}}^-)=0$$

$$A_n(1_q^+;2^+,\dots,n-1^+;n_{\bar{q}}^+)=\frac{m}{\beta_+\langle 1\,n\rangle}A_n(1_s;2^+,\dots,n-1^+;n_s)$$

- ➤ fermionic amplitude interesting by its own
- ➤ scalar amplitude of use in the unitarity method
- ➤ **building block for other helicity configurations**

Conclusions

- **QCD** is the toolkit for discovering new physics at hadron colliders
  - at least one-loop (NLO), NNLO better
  - and many legs
  - resummations beyond NLL
- **Twistor** inspired methods may provide an important phenomenological impact

  ⚠ still integration over ***phase-space*** non-trivial, and quite CPU time consuming

BACKUP

Heavy fermions

G.Rodrigo, JHEP **0509** (2005) 042

Spinors for massive fermions can be constructed from two light-like vectors

$$p_1^\mu = \beta_+ \hat{p}_1^\mu + \beta_- \hat{p}_2^\mu$$
$$p_1^\mu = \beta_- \hat{p}_1^\mu + \beta_+ \hat{p}_2^\mu \qquad \beta_\pm = (1 \pm \beta)/2 \qquad \beta = \sqrt{1 - 4m^2/s_{12}}$$

preserves **momentum conservation.**
The corresponding spinors are

$$\bar{u}_\pm(p_1, m) = \frac{\beta_+^{-1/2}}{\langle 2^\pm | 1^\mp \rangle} \langle 2^\mp | (\not{p}_1 + m)$$
$$v_\pm(p_2, m) = \frac{\beta_+^{-1/2}}{\langle 2^\pm | 1^\mp \rangle} (\not{p}_2 - m) | 1^\pm \rangle$$

and in the massless limit

$$\bar{u}_\pm(p_1) = \langle 1^\pm | \qquad v_\pm(p_2) = | 2^\mp \rangle$$