Effects on Fluctuations and Correlations of the Clustering of Color Sources

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1. **Transverse momentum fluctuations**
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WHY EVENT-BY-EVENT FLUCTUATIONS?

Non-statistical event-by-event fluctuations in relativistic heavy ion collisions has been proposed as as **probe of phase instabilities near de QCD phase transition.**

The fluctuations of the mean transverse momentum or mean multiplicity are related to the fundamental properties of the system, so may reveal information about the **QCD phase boundary.**

A phase transition in the evolution of the system created in relativistic heavy ion collisions may lead to a **divergence of the specific heat** which could be observed as **event-by-event fluctuations.**

EVENT-BY-EVENT P_T FLUCTUATIONS

Event-by-event fluctuations of p_T have been measured at SPS and RHIC

Behaviour of the non-statistical fluctuations as a function of the centrality of the collision:

- grow as the centrality increases

- maximum at mid centralities

- decrease at larger centralities

Different mechanisms have been proposed in order to explain those data:

- complete or partial equilibration

- critical phenomena

- production of jets

- string clustering or string percolation.

We are going to use:
CLUSTERING OF COLOR SOURCES

- Color strings are stretched between the projectile and target

- Strings = Particle sources: particles are created via sea $q\bar{q}$ production in the field of the string

- Color strings = Small areas in the transverse space filled with color field created by the colliding partons

- With growing energy and/or atomic number of colliding particles, the number of sources grows

- So the elementary color sources start to overlap, forming clusters, very much like disk in the 2-dimensional percolation theory

- In particular, at a certain critical density, a macroscopic cluster appears, which marks the percolation phase transition

• So we try to introduce a phase transition ($\equiv$QGP?)

(N. Armesto et al., PRL77 (96); J.Dias de Deus et al., PLB491 (00); M. Nardi and H. Satz).

• How?: Strings fuse forming clusters. At a certain critical density η_c (central PbPb at SPS, central AgAg at RHIC, central SS at LHC) a macroscopic cluster appears which marks the percolation phase transition (second order, non thermal).
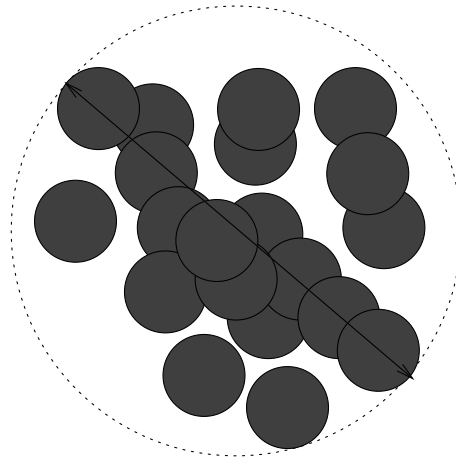
$$\eta = N_{st}\frac{S_1}{S_A} \ , \quad S_1 = \pi r_0^2, \quad r_0 = 0.2 \ \text{fm}, \quad \eta_c = 1.1 \div 1.2.$$

• Hypothesis: clusters of overlapping strings are the sources of particle production, and central multiplicities and transverse momentum distributions are little affected by rescattering.

- For a cluster of n overlapping strings covering an area S_n we calculate the multiplicity and p_T of the produced particles :

Color charge of the cluster=Vectorial sum of the strings charges

$$\vec{Q}_n = \sum_{i=1}^{n} \vec{Q}_{1i} \qquad \langle \vec{Q}_{1i} \cdot \vec{Q}_{1j} \rangle = 0 \qquad \vec{Q}_n^2 = n\vec{Q}_1^2$$

$$Q_n = \sqrt{\frac{nS_n}{S_1}}Q_1 \qquad \mu_n = \sqrt{\frac{nS_n}{S_1}}\mu_1 \qquad \langle p_T^2 \rangle_n = \sqrt{\frac{nS_1}{S_n}}\langle p_T^2 \rangle_1$$

For strings without interaction:

$$S_n = nS_1 \qquad Q_n = nQ_1 \qquad \Longrightarrow \qquad \mu_n = n\mu_1 \qquad \langle p_T^2 \rangle_n = \langle p_T^2 \rangle_1$$

For strings with max overlapping:

$$S_n = S_1 \qquad Q_n = \sqrt{n}Q_1 \qquad \Longrightarrow \qquad \mu_n = \sqrt{n}\mu_1 \qquad \langle p_T^2 \rangle_n = \sqrt{n}\langle p_T^2 \rangle_1$$

IN THE CLUSTERING APPROACH:

The behaviour of the p_T fluctuations can be understood as follows:

- **At low density**: most of the particles are produced by individual strings with the same $< p_T >_1$

  $\Rightarrow$ **fluctuations are small**

- **At large density** above the critical point: only one cluster

  $\Rightarrow$ **fluctuations are not expected either** *"equilibration"*

- **Just below the percolation critical density**: Large number of clusters formed by different number of strings, different size and different $< p_T >_n$

  $\Rightarrow$ **fluctuations are maximal**

Variables to measure event-by-event p_T fluctuations

F_{p_T} quantifies the deviation of the observed fluctuations from statistically independent particle emission

$$F_{p_T} = \frac{\omega_{data} - \omega_{random}}{\omega_{random}}, \quad \omega = \frac{\sqrt{<p_T^2> - <p_T>^2}}{<p_T>}$$

$$\phi = \sqrt{\frac{<Z^2>}{<\mu>}} - \sqrt{< z^2 >}$$

$z_i = p_{Ti} - < p_T >$ is defined for each particle

$Z_i = \sum_{j=1}^{N_i} z_j$ is defined for each event

$$F_{p_T} = \frac{\phi}{\sqrt{<z^2>}} = \frac{1}{\sqrt{<z^2>}} \sqrt{\frac{<Z^2>}{<\mu>}} - 1$$

- Mean cluster multiplicity and mean cluster p_T:

$$< \mu >_n= \sqrt{\frac{nS_n}{S_1}} < \mu >_1, \quad < p_T >_n= \left(\frac{nS_1}{S_n}\right)^{1/4} < p_T >_1$$

where $< \mu >_1$ and $< p_T >_1$ correspond to the mean multiplicity and the mean transverse momentum of the particles produced by one individual string.

- In order to obtain the mean p_T and the mean multiplicity of the collision at a given centrality:sum over all clusters and average over all events:

$$< \mu >= \frac{\sum_{i=1}^{N_{events}} \sum_j < \mu >_{n_j}}{N_{events}}, \quad < p_T >= \frac{\sum_{i=1}^{N_{events}} \sum_j < \mu >_{n_j}< p_T >_{n_j}}{\sum_{i=1}^{N_{events}} \sum_j < \mu >_{n_j}}$$

• Introducing our formula for the multiplicity of the cluster μ_{n_j} and the mean momentum $< p_T >_{n_j}$ we get:

$$< p_T >= \frac{\sum_{i=1}^{N_{events}} \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1 \left(\frac{n_j S_1}{S_{n_j}}\right)^{1/4} < p_T >_1}{\sum_{i=1}^{N_{events}} \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1}$$

• For the quantities $< z^2 >$ and $< Z^2 >$ we obtain:

$$< z^2 >= \frac{\sum_{i=1}^{N_{events}} \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1 \left[\left(\frac{n_j S_1}{S_{n_j}}\right)^{1/4} <p_T>_1 - <p_T>\right]^2}{\sum_{i=1}^{N_{events}} \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1}$$

and

$$\frac{< Z^2 >}{< \mu >} = \frac{\sum_{i=1}^{N_{events}} \left[\sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1 \left[\left(\frac{n_j S_1}{S_{n_j}}\right)^{1/4} <p_T>_1 - <p_T>\right]\right]^2}{\sum_{i=1}^{N_{events}} \sum_j \left(\frac{n_j S_{n_j}}{S_1}\right)^{1/2} \mu_1}$$

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$$F_{p_T} = \frac{\phi}{\sqrt{< z^2 >}} = \frac{1}{\sqrt{< z^2 >}}\sqrt{\frac{< Z^2 >}{< \mu >} - 1}$$

In order to compute F_{p_T} we need:

- A **Monte Carlo code for the cluster formation**, in order to compute the number of strings that come into each cluster and the area of the cluster

- We do not use a Monte Carlo code for the decay of the cluster, since we apply **analytical expressions for the transverse momentum and the multiplicities** of the clusters

- We also need **the value of** μ_1 –multiplicity produced by one individual string–. The total multiplicity per unit rapidity produced by one string has been taken as $\mu_{0\ tot} \simeq 1$
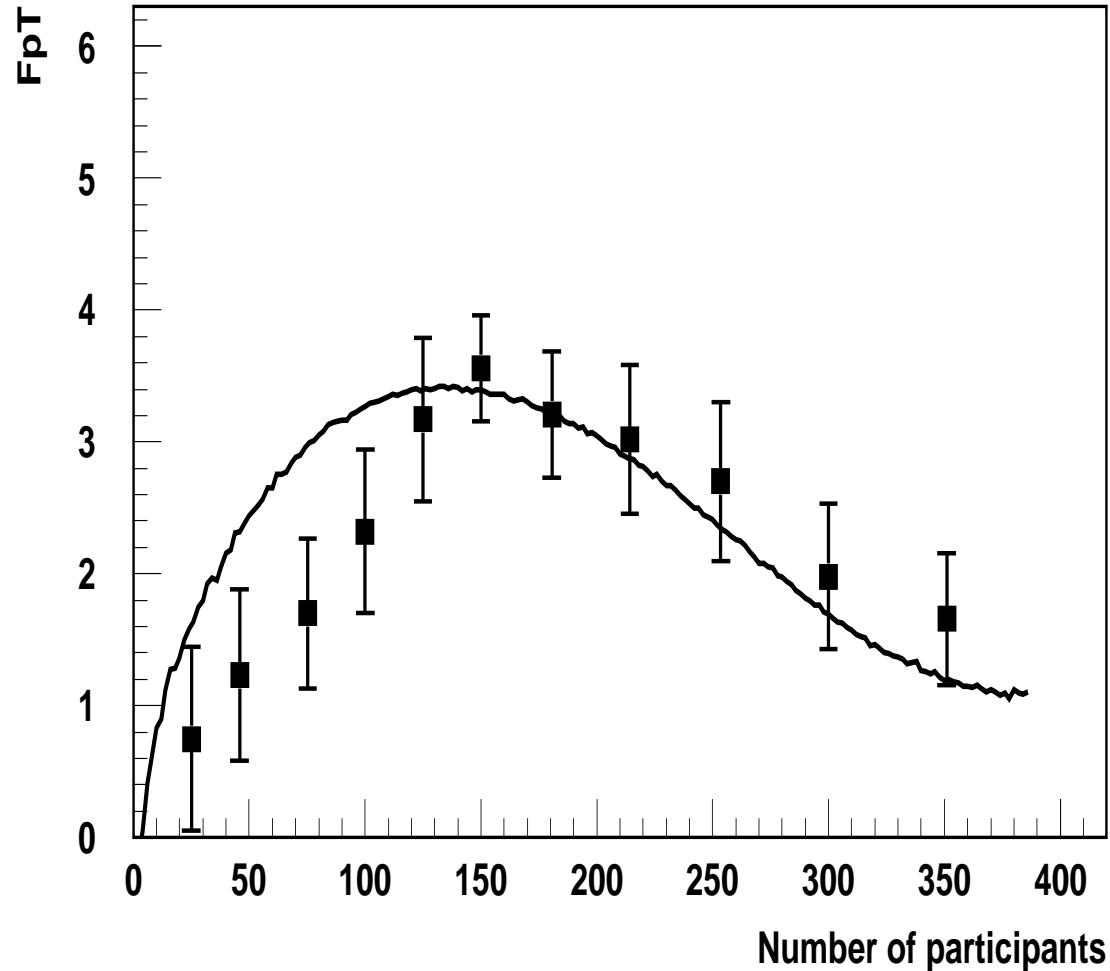
FLUCTUATIONS AT RHIC

Figure 1: $F_{p_T}(\%)$ versus the number of participants. Experimental data from PHENIX at $\sqrt{s} = 200$ GeV are compared with our results (solid line).
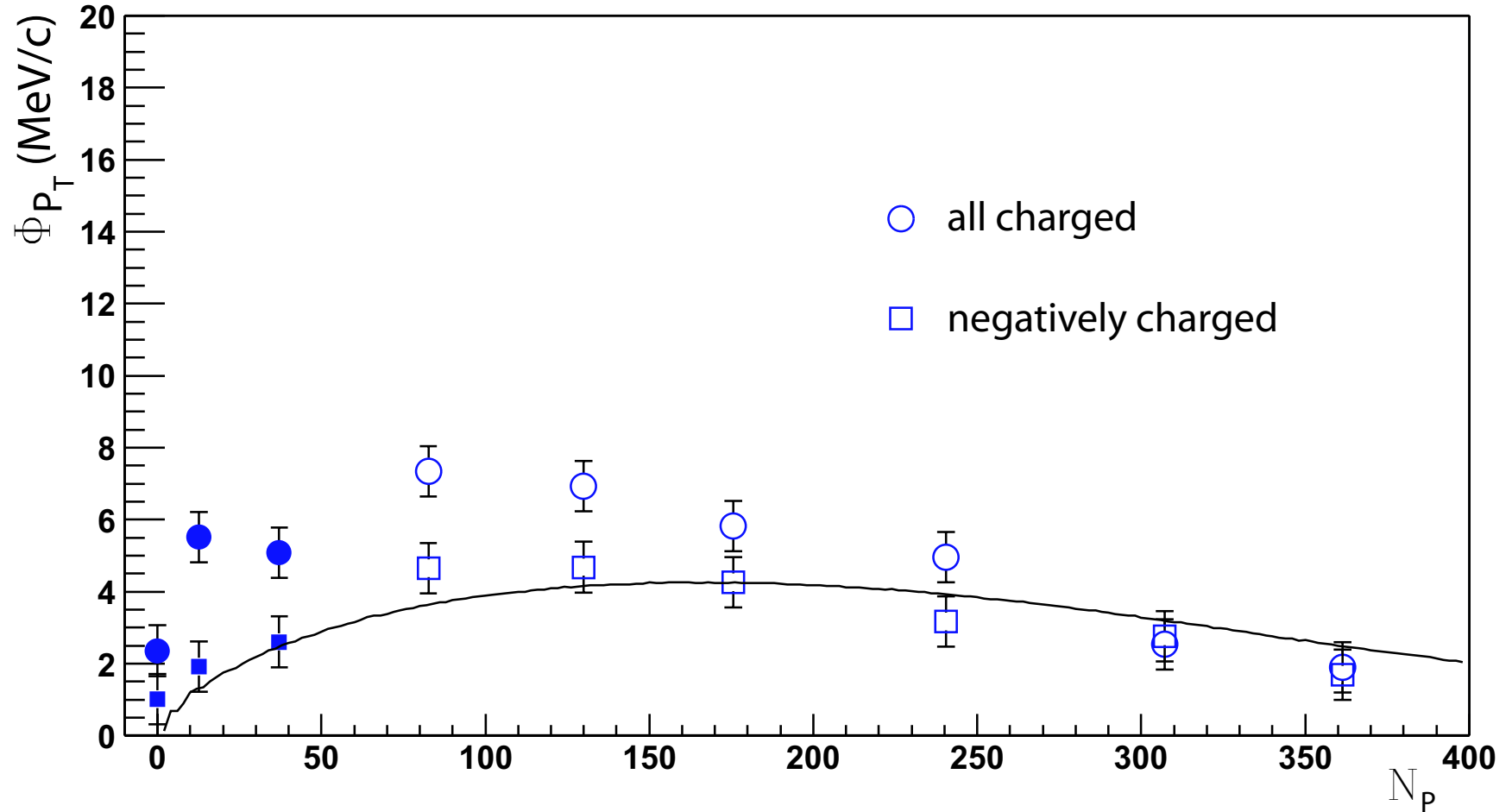
FLUCTUATIONS AT SPS

Figure 2: ϕ_{p_T} versus the number of participants. Experimental data from NA49 Collaboration at SPS energies are compared with our results (solid line).

• The **behaviour of the transverse momentum fluctuations** with the centrality of the collision shown by the RHIC data is naturally explained by the **clustering of color sources.**

• In this framework, elementary color sources −$strings$− **overlap** forming clusters, so the number of effective sources is modified.

• These clusters decay into particles with mean transverse momentum that depends on the **number of elementary sources** that conform each cluster, and the **area** occupied by the cluster.

• The **transverse momentum fluctuations** in this approach correspond to the fluctuations of the transverse momentum of these clusters, and they behave essentially as the **number of effective sources**.

• **In a jet production scenario** the mean p_T fluctuations are attributed to jet production in peripheral events, combined with jet suppression at larger centralities.

• **A way to discriminate between the two approaches:** fluctuations at SPS energies, where jet production cannot play a fundamental role.

EVENT-BY-EVENT MULTIPLICITY FLUCTUATIONS

• Recently, the NA49 Collaboration have presented their data on **multiplicity fluctuations as a function of centrality at SPS energies**.

• The variance of the multiplicity distribution scaled to the mean value of the multiplicity has been used.

$$Var(N) = \frac{<N^2> - <N>^2}{<N>}$$

A non-monotonic centrality –system size– dependence was found.

• **Its behaviour is similar to the one obtained for $\Phi(p_T)$** –used by the NA49 Collaboration to quantify the p_T-fluctuations– $\Longrightarrow$ **they are related to each other.**

• The Φ-measure is independent of the distribution of number of particle sources if the sources are identical and independent from each other

Our formula for the scaled variance obeys:

$$\frac{Var(\mu)}{<\mu>} = 1 + <\mu>_1 \frac{\left\langle \left(\sum_j \sqrt{\frac{n_j S_{n_j}}{S_1}}\right)^2 \right\rangle - \left\langle \sum_j \sqrt{\frac{n_j S_{n_j}}{S_1}} \right\rangle^2}{\left\langle \sum_j \sqrt{\frac{n_j S_{n_j}}{S_1}} \right\rangle} ,$$

In order to obtain the scaled variance we have calculated $<\mu^2>$:

$$<\mu^2> = \frac{1}{N_{events}}\left[\sum_{i=1}^{N_{events}} \left(\sum_j \sqrt{\frac{n_j S_{n_j}}{S_1}}\right)^2 <\mu>_1^2 + \sum_{i=1}^{N_{events}} \sum_j \sqrt{\frac{n_j S_{n_j}}{S_1}} <\mu>_1 \right]$$

where we have supposed that the multiplicity of each cluster follows a Poissonian of mean value $<\mu>_{n_j}$, $<\mu^2>_{n_j} = <\mu>_{n_j}^2 + <\mu>_{n_j}$.

Behaviour of the scaled varianza

- **Low density limit** –isolated strings that do not interact–:

$$\frac{Var(\mu)}{<\mu>} = 1 + <\mu>_1 \frac{<N_s^2> - <N_s>^2}{<N_s>} \simeq 1 + <\mu>_1$$

where N_s corresponds to the number of strings that, for a fixed number of participants: $\frac{<N_s^2> - <N_s>^2}{<N_s>} \simeq 1$ (Poissonian distribution).

- **In the large density regime** –all the strings fuse into a single cluster that occupies the whole interaction area–:

$$\frac{Var(\mu)}{<\mu>} = 1 + <\mu>_1 \frac{\left\langle \left(\sqrt{\frac{N_s S_A}{S_1}}\right)^2 \right\rangle - \left\langle \sqrt{\frac{N_s S_A}{S_1}} \right\rangle^2}{\left\langle \sqrt{\frac{N_s S_A}{S_1}} \right\rangle} \simeq 1$$

where S_A is the nuclear overlap area.

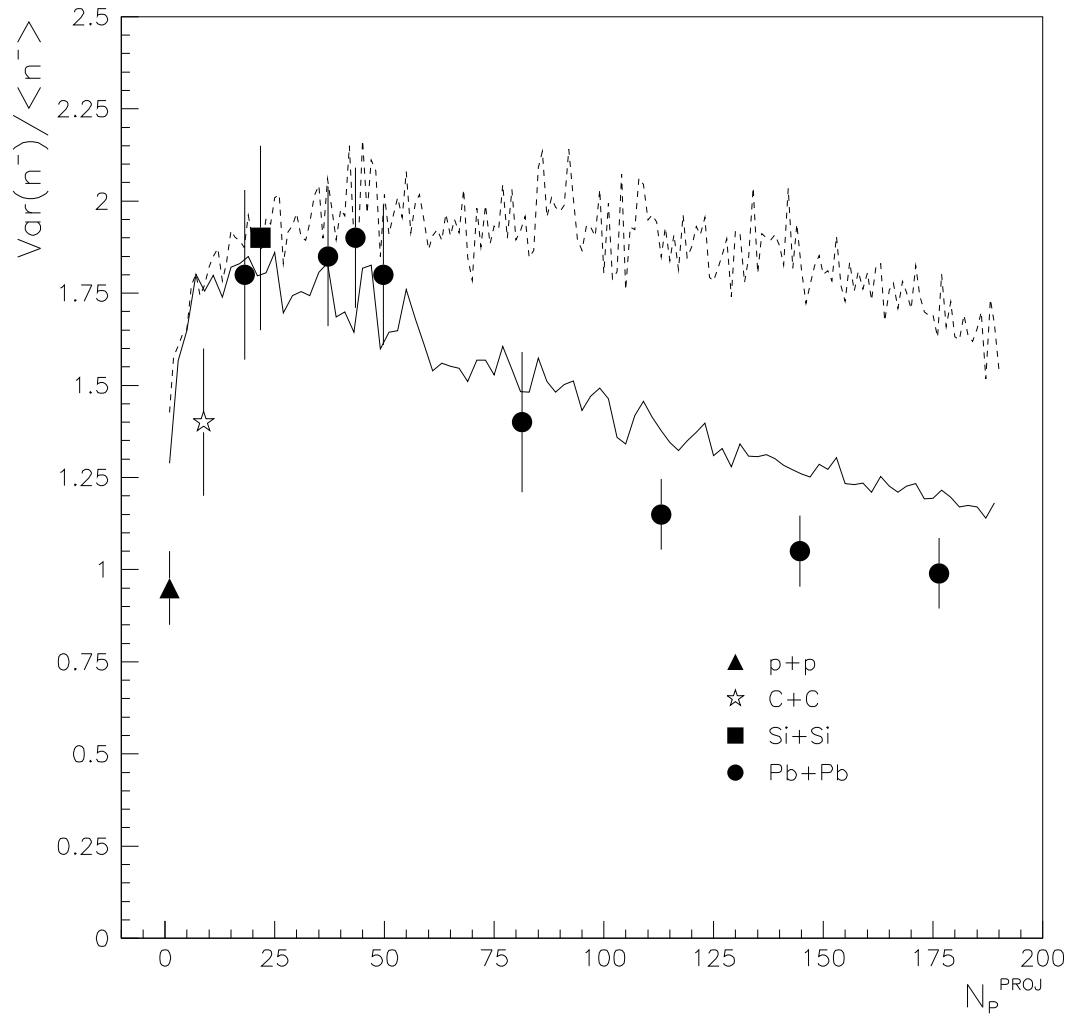The second element of the r.h.s. of this equation tends to zero.

Figure 3: Our results for the scaled variance of negatively charged particles in Pb+Pb collisions at $P_{lab} =$158 AGeV/c compared to NA49 experimental data. The dashed line corresponds to our result when clustering formation is not included, the continuous line takes into account clustering.

Figure 4: Our results for the variance of negatively charged particles in Pb+Pb collisions at $P_{lab} =$158 AGeV/c compared to NA49 experimental data. The dashed line corresponds to our result when clustering formation is not included, the continuous line takes into account clustering.
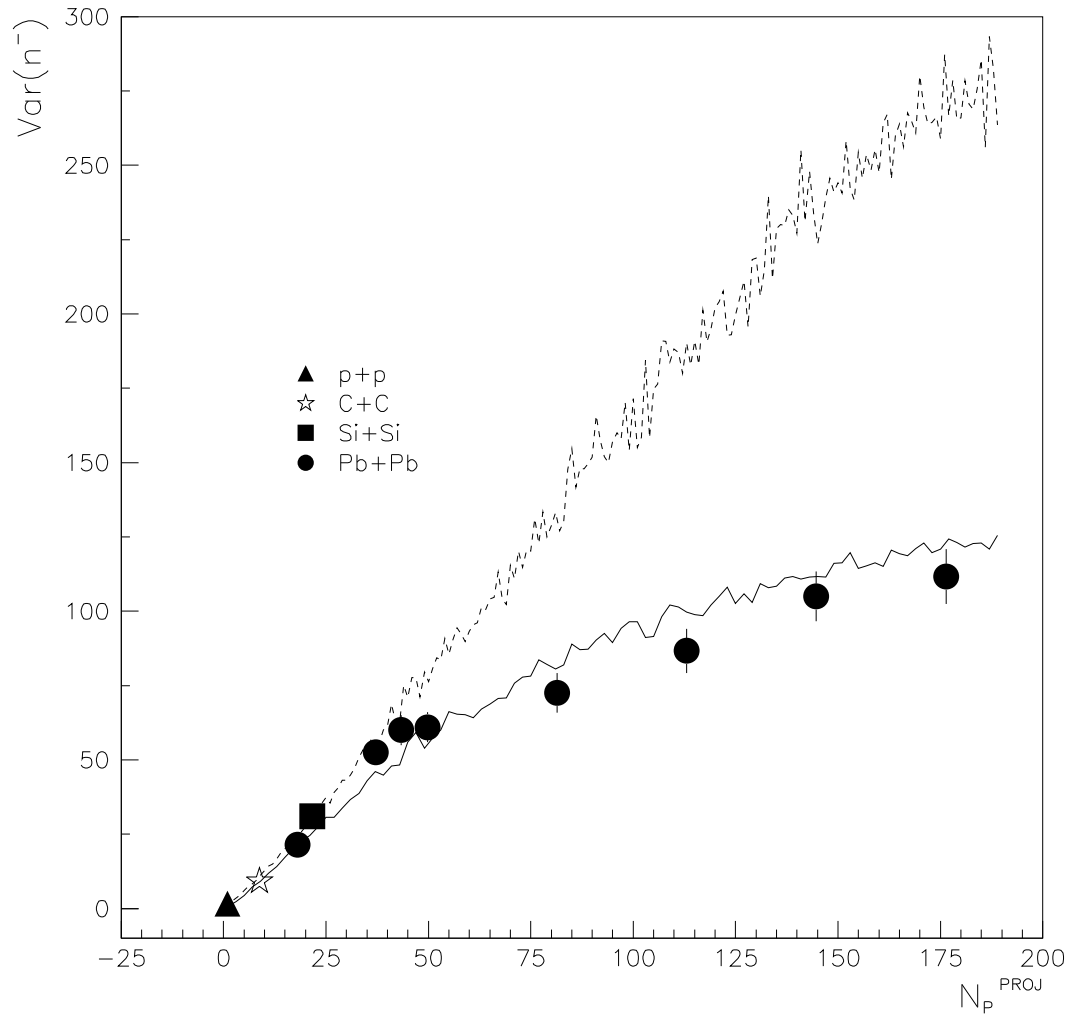
Figure 5: Our results for the mean multiplicity of negatively charged particles in Pb+Pb collisions at $P_{lab} =$158 AGeV/c compared to NA49 experimental data. The dashed line corresponds to our result when clustering formation is not included, the continuous line takes into account clustering.
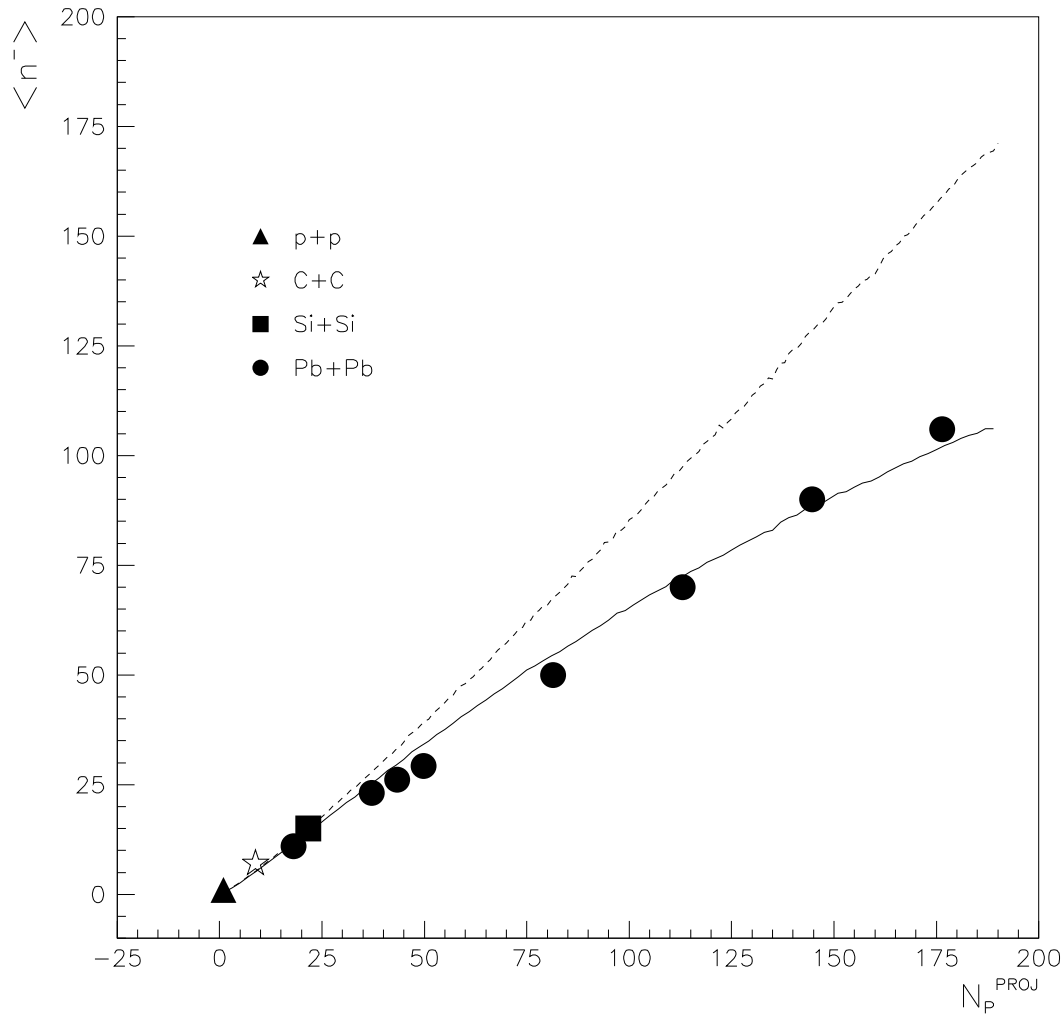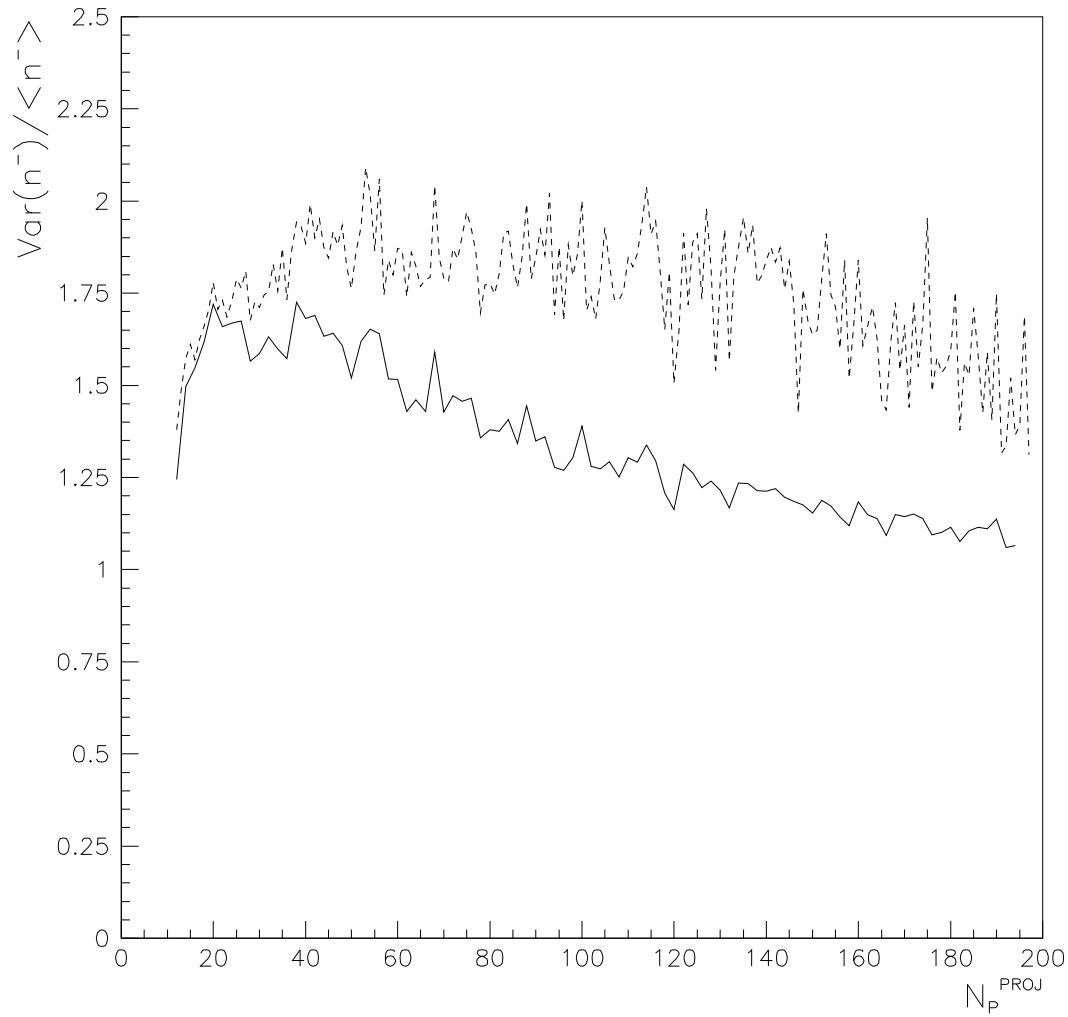
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Figure 6: Our results for the scaled variance of negatively charged particles in Au+Au collisions at $\sqrt{s} = 200$ GeV. The dashed line corresponds to our result when clustering formation is not included, the continuous line takes into account clustering.

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• We find a non-monotonic dependence of the multiplicity fluctuations with the number of participants.

The centrality behaviour of these fluctuations is very similar to the one found for the mean p_T fluctuations.

• In our approach, the mechanism responsible for multiplicity and mean p_T fluctuations is the formation of clusters of strings that introduces correlations between the produced particles.

• On the other hand, p_T fluctuations have been attributed to jet production in peripheral events, combined with jet suppression in central events.

• However, this hard-scattering interpretation can not be applied to SPS energies, so it does not explain the non-monotonic behaviour of the mean p_T fluctuations neither the relation between mean p_T and multiplicity fluctuations at SPS energy.

• Other possible mechanisms are: combination of strong and electromagnetic interaction, dipole-dipole interaction and non-extensive thermodynamics. Still, it is not clear if these fluctuations have a kinematic or dynamic origin, but clustering of colour sources remains a good possibilty.

LONG AND SHORT RANGE CORRELATIONS

- A measurement of such correlations is the backward–forward dispersion

$$D^2_{BF} =< n_B\ n_F > - < n_B >< n_F >$$

where n_B (n_F) is the number of particles in a backward (forward) rapidity

- Cluster formation implies the fusion of sources close in rapidity

$\implies$ The long range correlations are not affected

$\implies$ The short range correlations are increased

- A measure of the clustering:

$$b = D^2_{BF}/D^2_{FF}$$

LONG AND SHORT RANGE CORRELATIONS AT SPS

Reaction	Regions		b	
	$B \equiv$	$F \equiv$		
pp at 45 GeV, 10000 events	$\eta < -1$	$\eta > 1$	NF	0.13
			F	0.11
			Exp	0.137 ± 0.023
$\overline{p}p$ at 540 GeV, 50000 events	$\eta < -1$	$\eta > 1$	NF	0.55
			F	0.53
	$-4 \leq \eta \leq -1$	$1 \leq \eta \leq 4$	Exp	0.41 ± 0.01
pp at 19.4 GeV, 10000 events	$0.75 < y < 1.75$	$3.25 < y < 4.25$	NF	0.04
			F	0.04
			Exp	-0.01 ± 0.01
pAr at 19.4 $AGeV$, 10000 events	$0.75 < y < 1.75$	$3.25 < y < 4.25$	NF	0.33
			F	0.33
			Exp	0.28 ± 0.04
pXe at 19.4 $AGeV$, 10000 events	$0.75 < y < 1.75$	$3.25 < y < 4.25$	NF	0.37
			F	0.35
			Exp	0.41 ± 0.04

LONG AND SHORT RANGE CORRELATIONS AT SPS

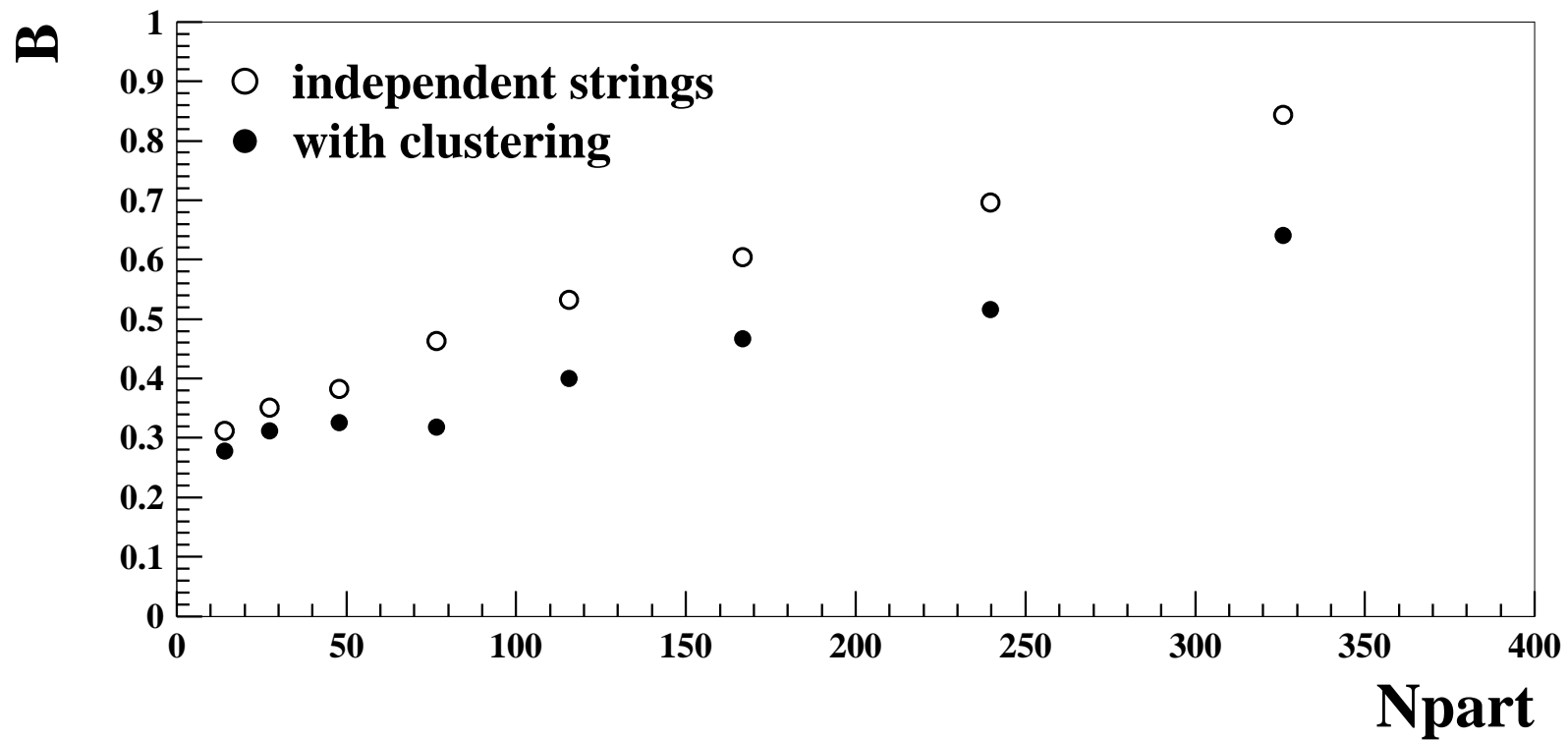|				D^2_{BF}	D^2_{FF}	$< n_F >$	b
$\sqrt{s_{NN}} =$ 19.4 GeV	SPb	$n_F \geq 80$	$NOFUS$	161	146	100	1.11
			FUS	153	138	99	1.11
	$PbPb$	$n_F \geq 550$	$NOFUS$	421	930	594	0.45
			FUS	222	646	587	0.34
$\sqrt{s_{NN}} =$ 200 GeV	SS	$n_F \geq 210$	$NOFUS$	634	789	248	0.80
			FUS	446	606	241	0.74
		$n_F \geq 260$	$NOFUS$	253	341	282	0.74
			FUS	105	255	279	0.41
	$CuCu$	$n_F \geq 360$	$NOFUS$	6481	6477	480	0.93
			FUS	3844	4196	458	0.92
		$n_F \geq 600$	$NOFUS$	617	723	636	0.85
			FUS	103	266	620	0.39

Figure 7: **LONG AND SHORT RANGE CORRELATIONS AT RHIC**
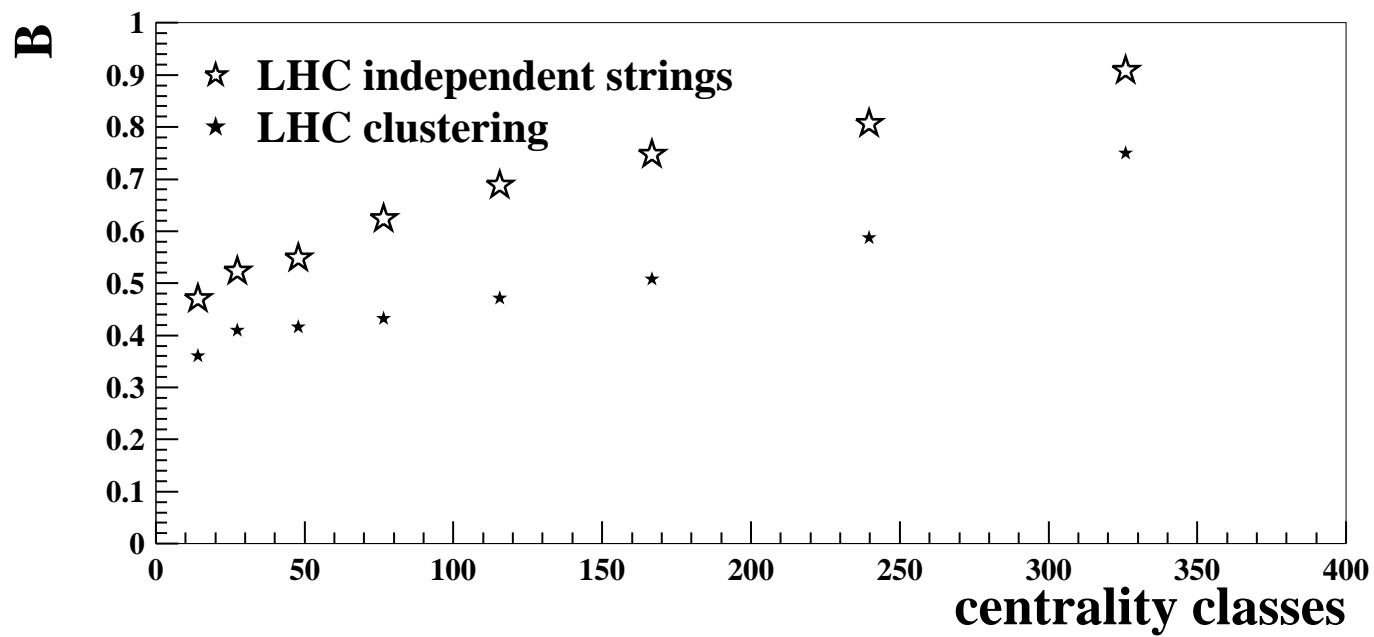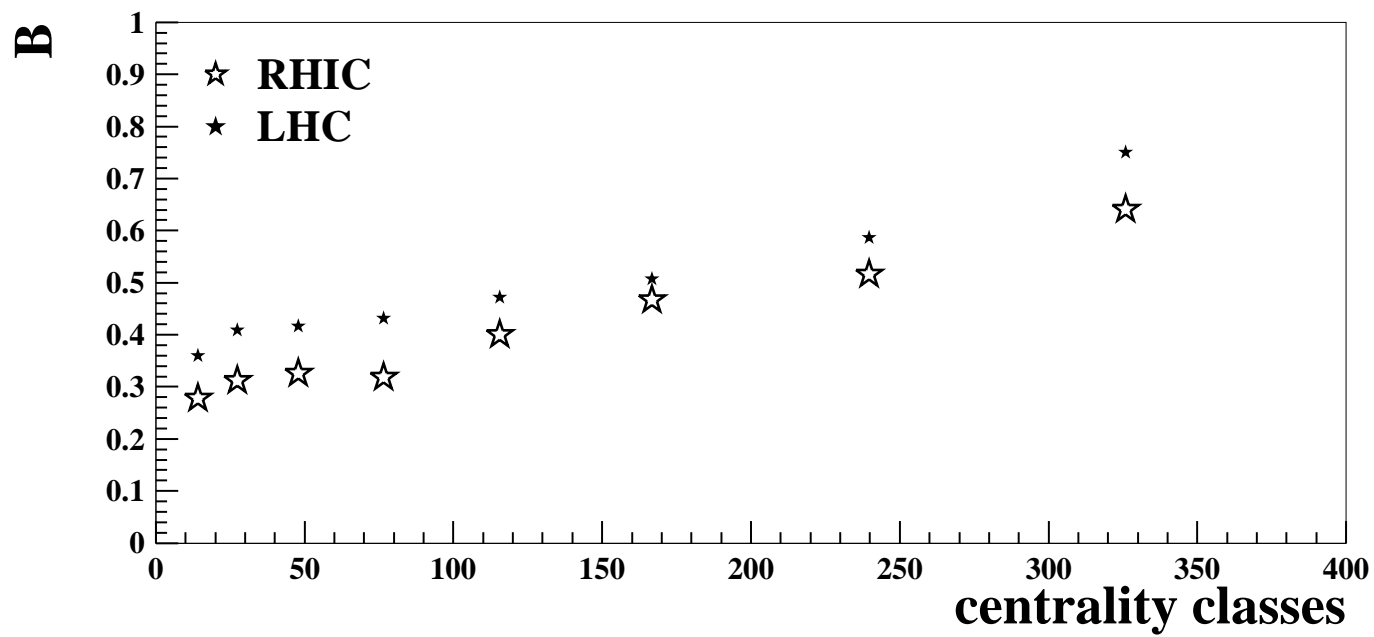
COMMENTS

- In order to compute our value for F_{p_T}, we take into account all possible transverse momenta, whereas in the experiment there is a limited acceptance, 0.2 GeV/c $< p_T < p_T^{max}$

- PHENIX has studied the variation of F_{p_T} with the maximal value of the acceptance for p_T, p_T^{max}. The maximum of F_{p_T} is reached for the largest acceptance, $p_T^{max} = 4$ GeV/c

- So we can expect that our value for F_{p_T} is going to be higher than the experimental one, specially for a moderate number of participants, N_p, since the truncated average p_T, $< p_T^{trunc} > = \dfrac{\int_{p_T^{min}}^{\infty} p_T dN/dp_T}{\int_{p_T^{min}}^{\infty} dN/dp_T} - p_T^{min}$, decreases with the number of participants for $p_T^{min} > 2$ GeV/c

- This means that, for momenta higher than 2 GeV/c, the high p_T contribution would be due to collisions with a moderate number of participants

- These considerations may explain the difference between our results and PHENIX data with a limited acceptance of $0.2 < p_T < 2$ GeV at low N_p

- In order to have a better understanding of the behaviour of F_{p_T} and ϕ_{p_T} on the number of participants, we plot in Fig. 3 and Fig. 4 the mean number of clusters M and the dispersion on the number of clusters multiplied by the number of clusters $\sigma_M * M$ at RHIC and SPS energies.

- The p_T fluctuations are due in our approach to the different mean transverse momenta of the clusters. These momenta depend on the number of strings that comes into the cluster and the area occupied by the cluster, therefore M and σ_M should be the key quantities.

- The only effect of σ_M is to shift the maximum of M. Because of this we expect the dependence of F_{p_T} and ϕ_{p_T} on N_p to be more similar to the M behaviour, as it is actually.

- In other words, a decrease in the number of effective sources leads to a decrease of the tranverse momentum fluctuations.
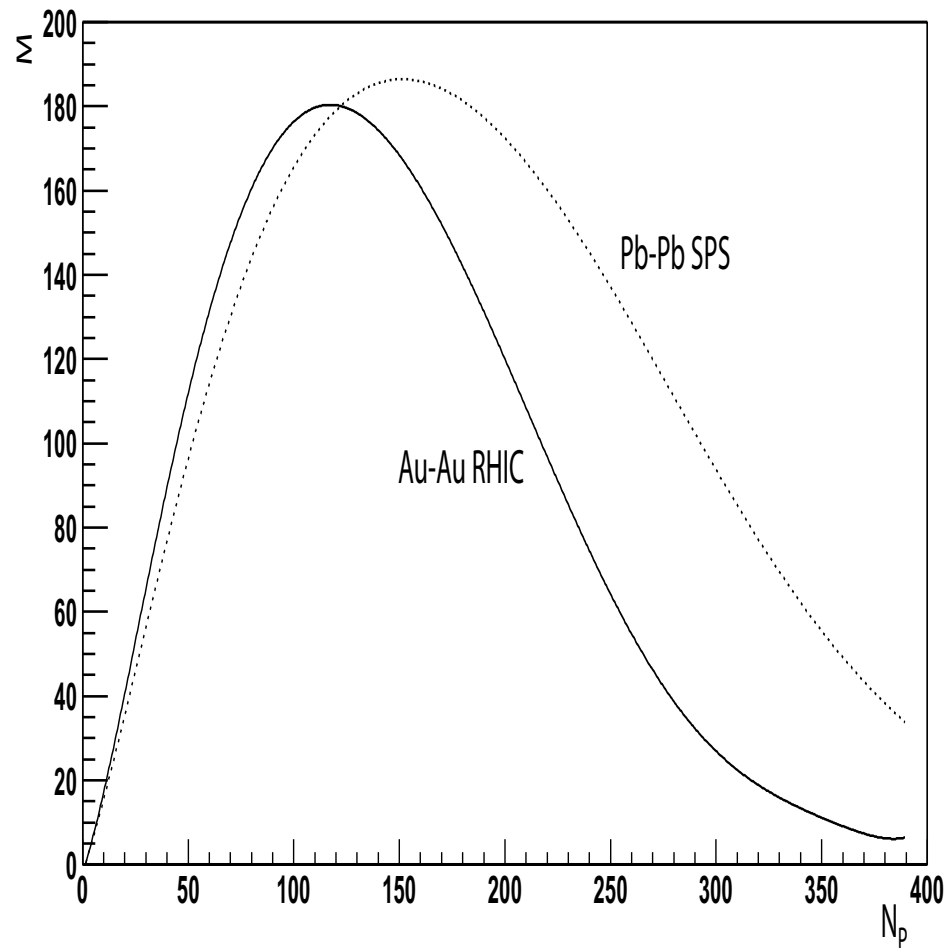
Figure 8: Mean number of clusters M versus the number of participants for Pb-Pb collisions at SPS energies (dotted line) and Au-Au collisions at RHIC energies (solid line).
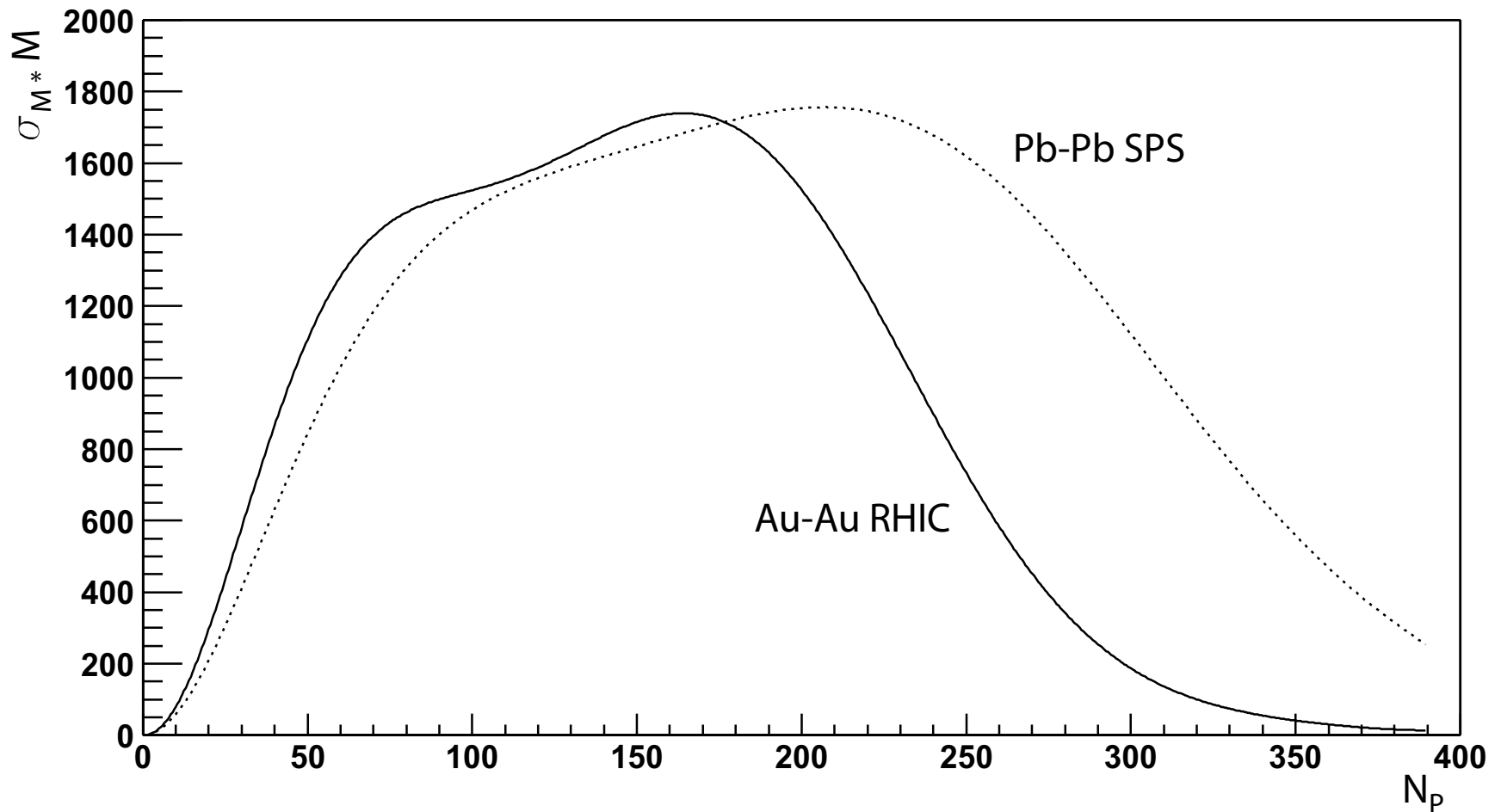
Figure 9: Dispersion on the number of clusters multiplied by the number of clusters $\sigma_M * M$ versus the number of participants for Pb-Pb collisions at SPS energies (dotted line) and Au-Au collisions at RHIC energies (solid line).