Elliptic flow results from parton transport

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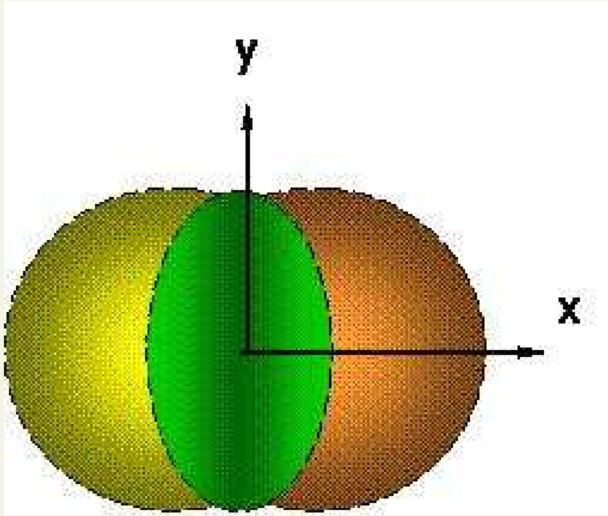
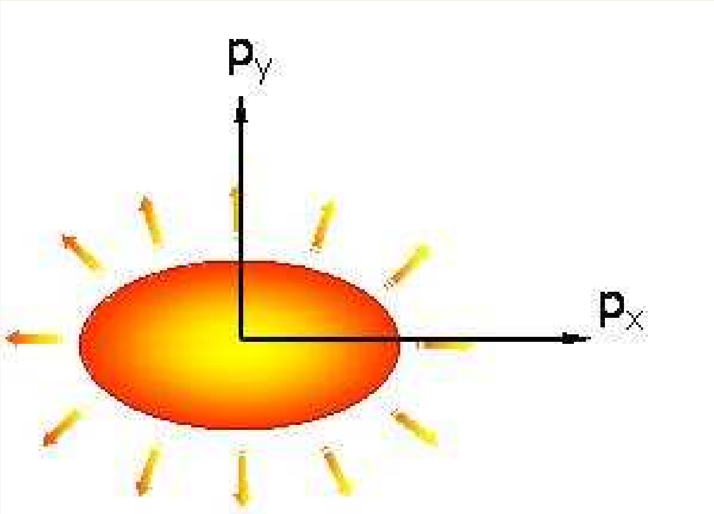
- **remarkable collectivity in A+A collisions - “elliptic flow”** (v_2)



$\longrightarrow$



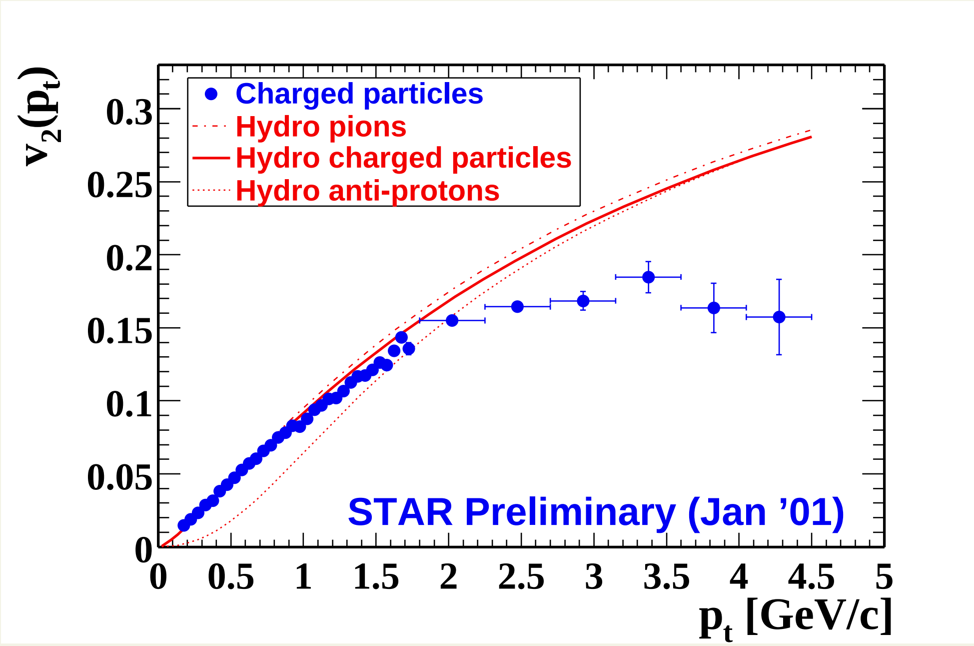
$$\varepsilon \equiv \frac{\langle x^2 - y^2 \rangle}{\langle x^2 + y^2 \rangle}$$

$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}$$

observed in A+A at (essentially) all energies

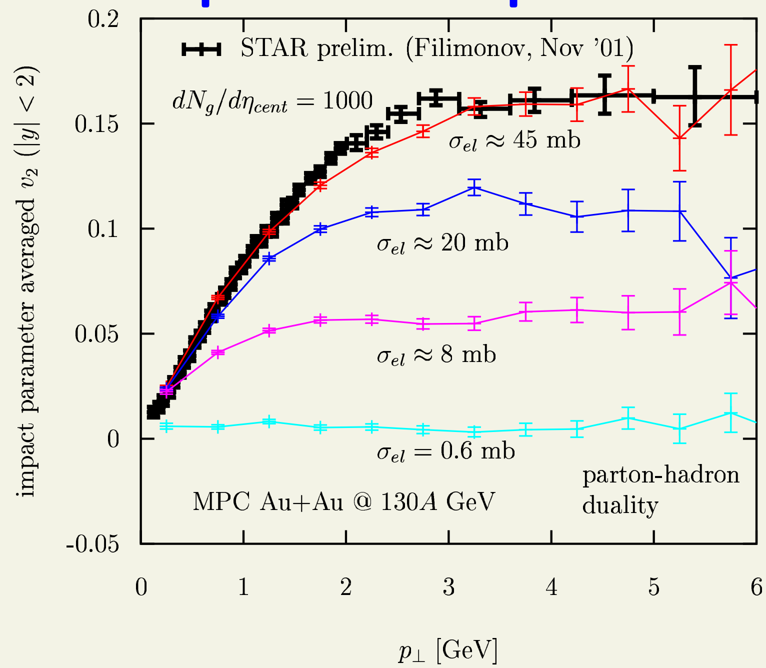
Kolb, Heinz et al

ideal hydrodynamics



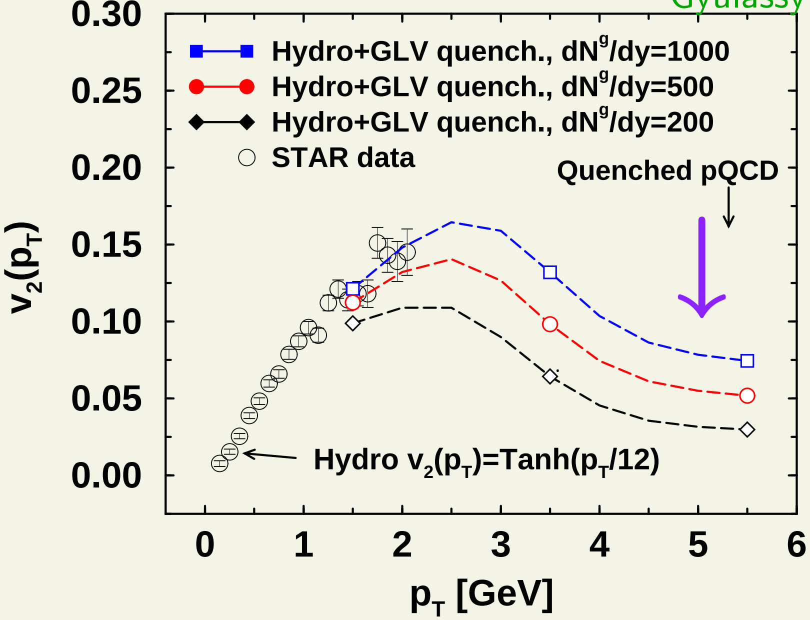
DM & Gyulassy

covariant parton transport



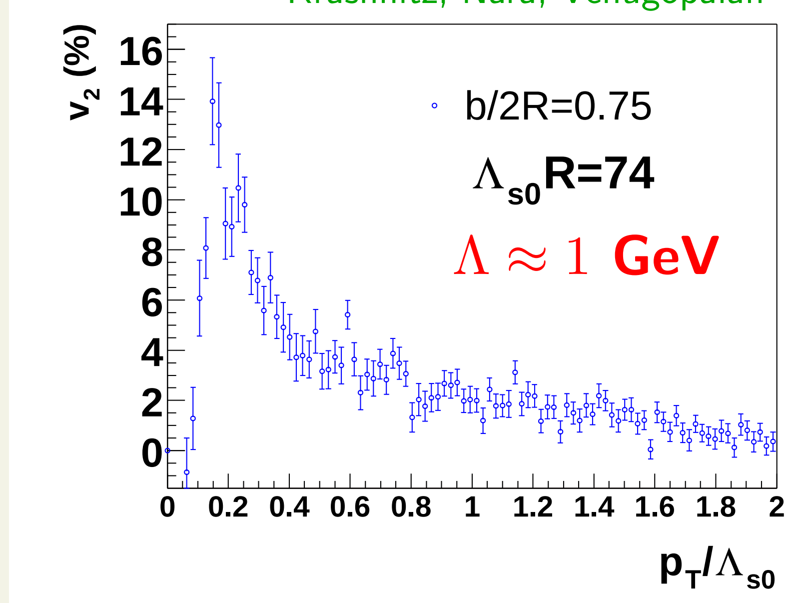
parton energy loss...

Gyulassy & Vitev



classical Yang-Mills ...

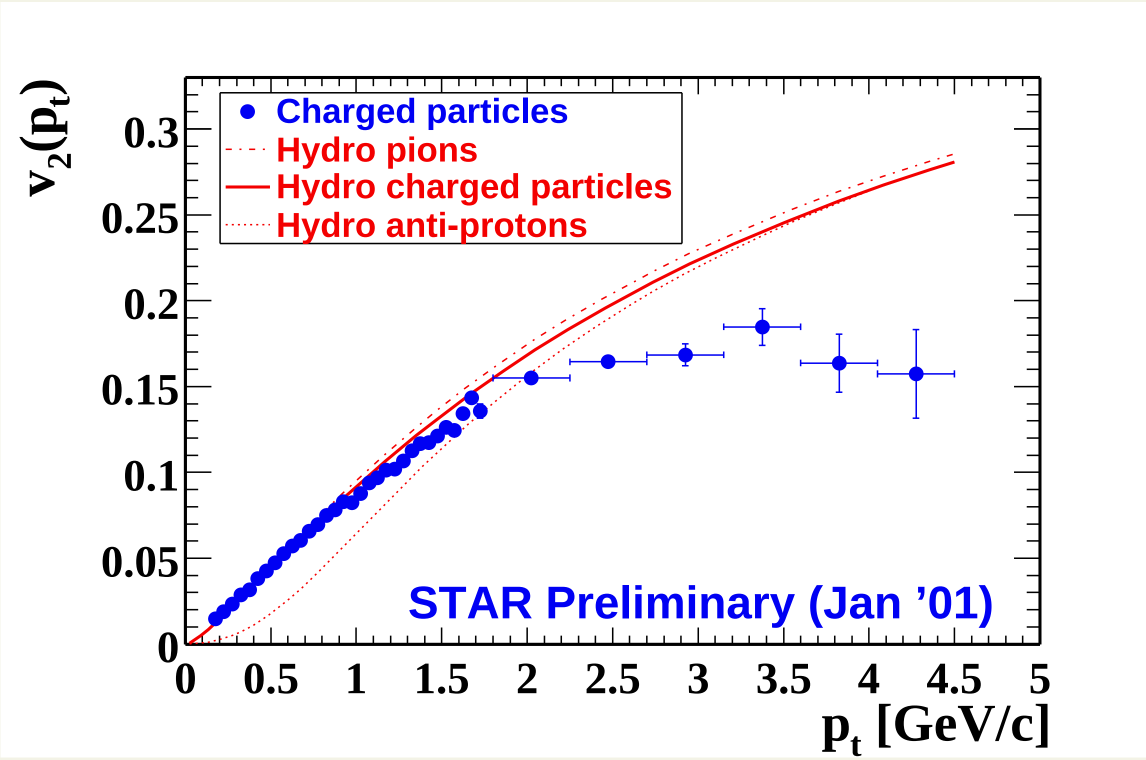
Krashnitz, Nara, Venugopalan

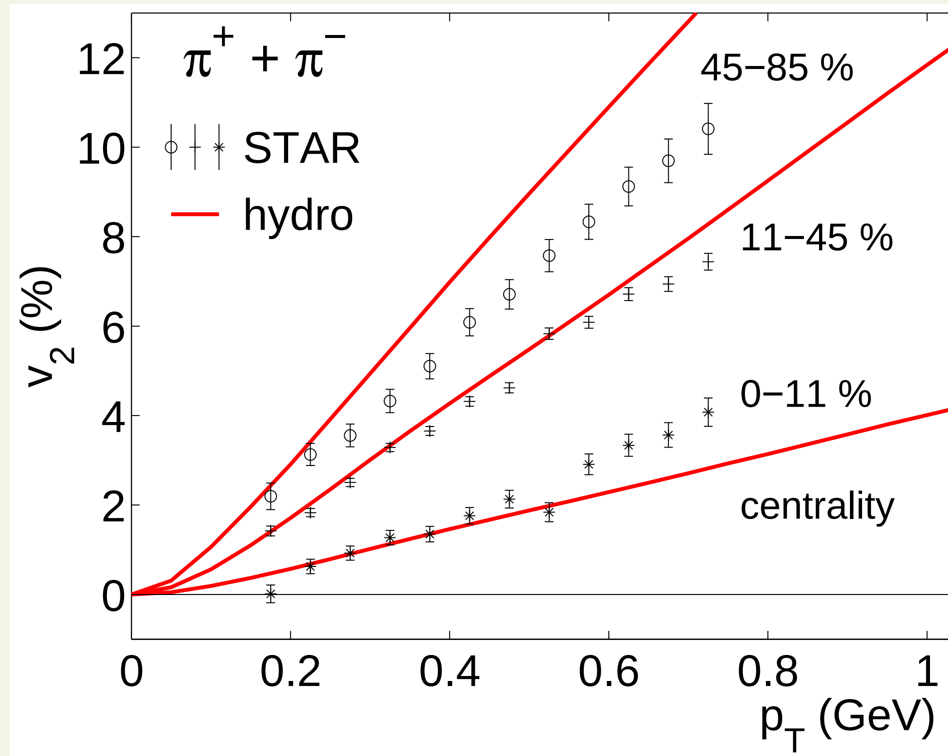


one explanation: thermalized matter with vanishing viscosity

works for minbias, below $p_T < 1.5$ GeV

but centrality dependence deviates already at p_T=0.7 GeV





deviations from local equilibrium give insights into thermalization mechanisms

$\Rightarrow$ **need a non-equilibrium approach**

Covariant transport

Boltzmann ..., Israel, Stewart, de Groot, ... Pang, Zhang, Gyulassy, DM, Vance, Csizmadia, Pratt, Cheng, Xu, Greiner ...

Covariant, causal, nonequil. approach - formulated in terms of local rates.

$$\Gamma_{2\to 2}(x) \equiv \frac{dN_{scattering}}{d^4x} = \sigma v_{rel} \frac{n^2(x)}{2}$$

transport eqn.: $f_i(\vec{x}, \vec{p}, t)$ - **phase space distributions**

$$p^\mu \partial_\mu f_i(\vec{x}, \vec{p}, t) = \overbrace{S_i(\vec{x}, \vec{p}, t)}^{\text{source}} + \overbrace{C_i^{el.}[f](\vec{x}, \vec{p}, t)}^{2 \to 2\ (\text{ZPC}, \text{GCP}, ...)} + \overbrace{C_i^{inel.}[f](\vec{x}, \vec{p}, t)}^{2 \leftrightarrow 3\ (\text{MPC}, \text{Xu}-\text{Greiner})} + ...$$

algorithms: OSCAR code repository @ http://nt3.phys.columbia.edu/OSCAR

HERE: utilize MPC algorithm DM, NPA 697 ('02)

rate is a local and manifestly covariant scalar

for particles with momenta p_1 and p_2:

$$\Gamma(x) = \sigma\, v_{rel}\, n_1(x) n_2(x) = \sigma\, \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{E_1 E_2}\, n_1(x) n_2(x)$$

(n/E is a scalar)

an equivalent alternative form is $v_{rel} = \sqrt{(\vec{v}_1 - \vec{v}_2)^2 - (\vec{v}_1 \times \vec{v}_2)^2}$

[in cascade algorithms, rate is evaluated in the pair c.o.m. frame, where $\vec{v}_1 || \vec{v}_2$ and thus $v_{rel} = |\vec{v}_1 - \vec{v}_2|$]

Example: Molnar's Parton Cascade

Elementary processes: elastic $2 \to 2$ processes + $gg \leftrightarrow q\bar{q}$, $q\bar{q} \to q'\bar{q}'$ + $ggg \leftrightarrow gg$

Equation for $f^i(x,\vec{p})$: $\quad i = \{g, d, \bar{d}, u, \bar{u}, \ldots\}$

$$
\begin{aligned}
p_1^\mu \partial_\mu \tilde{f}^i(x,\vec{p}_1) \quad = \quad & \frac{\pi^4}{2} \sum_{jkl} \int\!\!\!\int\!\!\!\int_{2\,3\,4} \left(\tilde{f}_3^k \tilde{f}_4^l - \tilde{f}_1^i \tilde{f}_2^j\right) \left|\bar{\mathcal{M}}_{12\to 34}^{i+j\to k+l}\right|^2 \delta^4(12-34) \qquad 2 \to 2 \\
& + \frac{\pi^4}{12} \int\!\!\!\int\!\!\!\int\!\!\!\int_{2\,3\,4\,5} \left(\frac{\tilde{f}_3^i \tilde{f}_4^i \tilde{f}_5^i}{g_i} - \tilde{f}_1^i \tilde{f}_2^i\right) \left|\bar{\mathcal{M}}_{12\to 345}^{i+i\to i+i+i}\right|^2 \delta^4(12-345) \qquad 2 \leftrightarrow 3 \\
& + \frac{\pi^4}{8} \int\!\!\!\int\!\!\!\int\!\!\!\int_{2\,3\,4\,5} \left(\tilde{f}_4^i \tilde{f}_5^i - \frac{\tilde{f}_1^i \tilde{f}_2^i \tilde{f}_3^i}{g_i}\right) \left|\bar{\mathcal{M}}_{45\to 123}^{i+i\to i+i+i}\right|^2 \delta^4(123-45) \qquad 3 \leftrightarrow 2 \\
& + \tilde{S}^i(x,\vec{p}_1) \quad \leftarrow \textbf{initial conditions}
\end{aligned}
$$

with shorthands:

$$
\tilde{f}_i^q \equiv (2\pi)^3 f_q(x,\vec{p}_i), \quad \int_i \equiv \int \frac{d^3 p_i}{(2\pi)^3 E_i}, \quad \delta^4(p_1+p_2-p_3-p_4) \equiv \delta^4(12-34)
$$

Hydrodynamic limit

mean free path: characterizes local conditions

$$\lambda(x) \equiv \frac{1}{\textbf{cross section} \times \textbf{density(x)}} \qquad \begin{cases} \lambda \ll \textit{size}, \textit{time} & -\text{ hydrodynamics} \\ \lambda \gg \textit{size}, \textit{time} & -\text{ free streaming} \end{cases}$$

- **Ideal fluid limit** $\lambda \to 0$: **local equilibrium** $f = (2\pi)^{-3} \exp[p_\mu u^\mu(x)/T(x)]$

  $$T^{\mu\nu}_{id} = (e+p)u^\mu u^\nu - p\, g^{\mu\nu}$$

  $\partial_\mu S^\mu = 0 \quad \Rightarrow$ **entropy conserved**

- **Viscous hydro** $\lambda \ll \textit{length \& time scales}$: **near equilibrium - slowly varying**

  dissipative dynamics in terms of transport coefficients and relaxation times

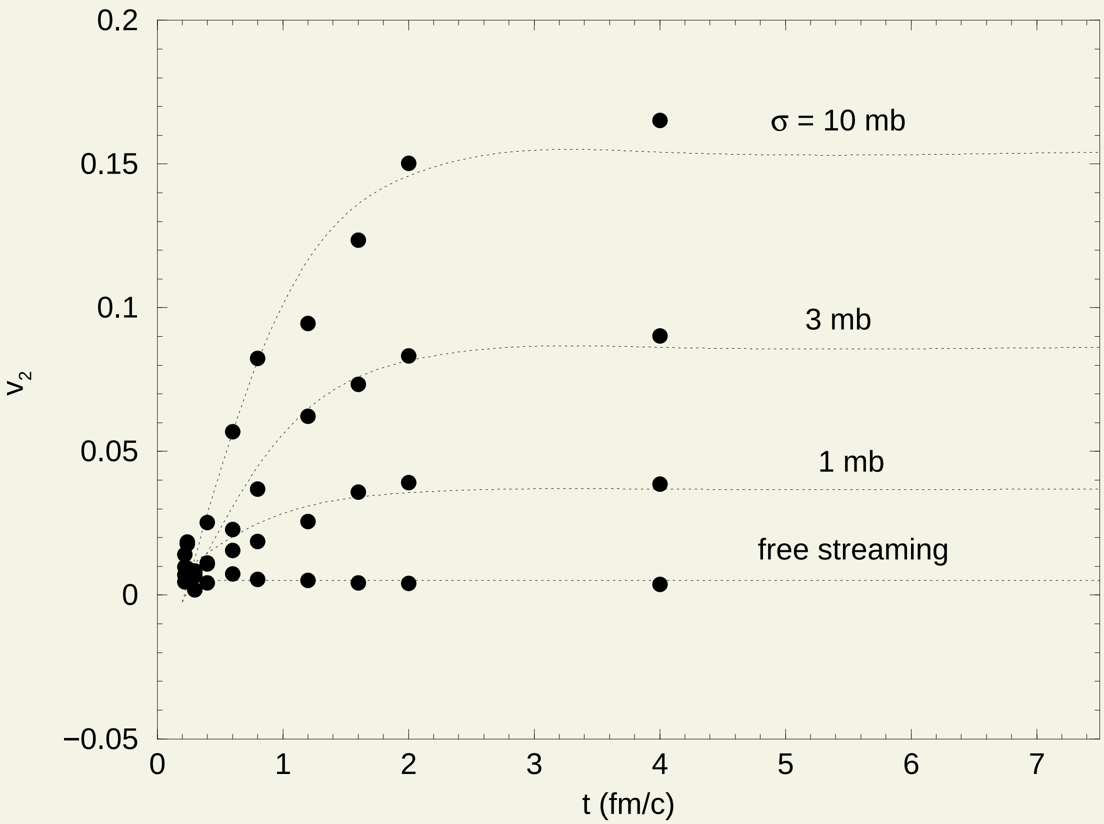
  $$e.g., \quad \text{shear viscosity } \eta \approx 0.8 \frac{T}{\sigma_{tr}}\,, \qquad \text{relaxation time } \tau_\pi \approx 1.2 \lambda_{tr}$$

but note: transport is ALWAYS causal Israel, Stewart ('79) ...

v_2 from transport

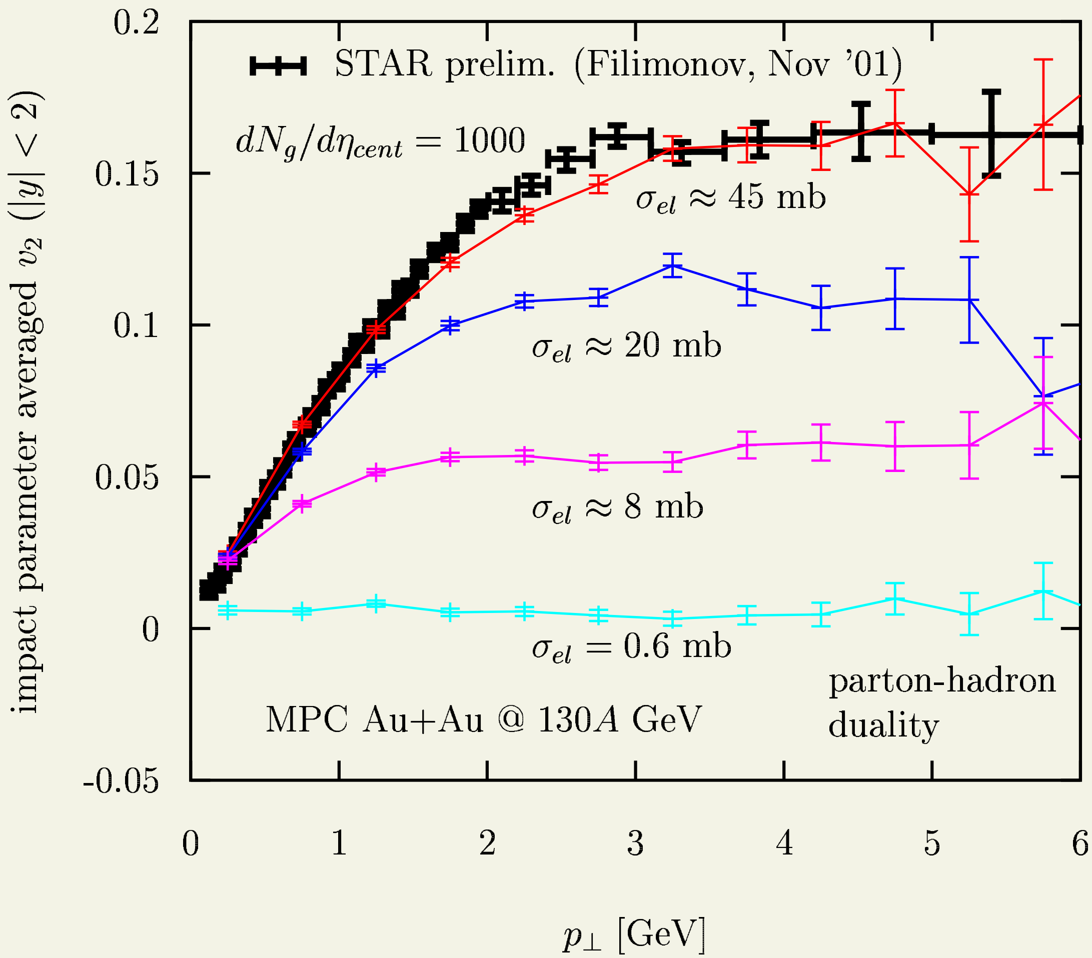
anisotropy increases with cross section, and developes early ($\sim 1 - 2$ fm$/c$)

Zhang, Gyulassy & Ko, PLB455 ('99): use ZPC algorithm



sharp cylinder $R = 5$ fm, $\tau_0 = 0.2$ fm$/c$, $b = 7.5$ fm, $dN^{cent}/dy = 300$

DM & Gyulassy, NPA 697 ('02): **$v_2(p_T, \chi)$ at RHIC**



parton transport model MPC

$2 \to 2$ only, forward-peaked

$\sigma_{tr} \approx 0.3\sigma_{tot}$

Au+Au @ 130 GeV, $b = 8$ fm

- HIJING (minijet+radiation) initconds
- $dN/d\eta$ based on EKRT saturation
- binary transverse profile
- 1 parton $\longrightarrow$ 1 π hadronization

RHIC: need $\sigma_{gg} \approx 45$ mb $\to 15\times$ perturbative $2 \to 2$ rates

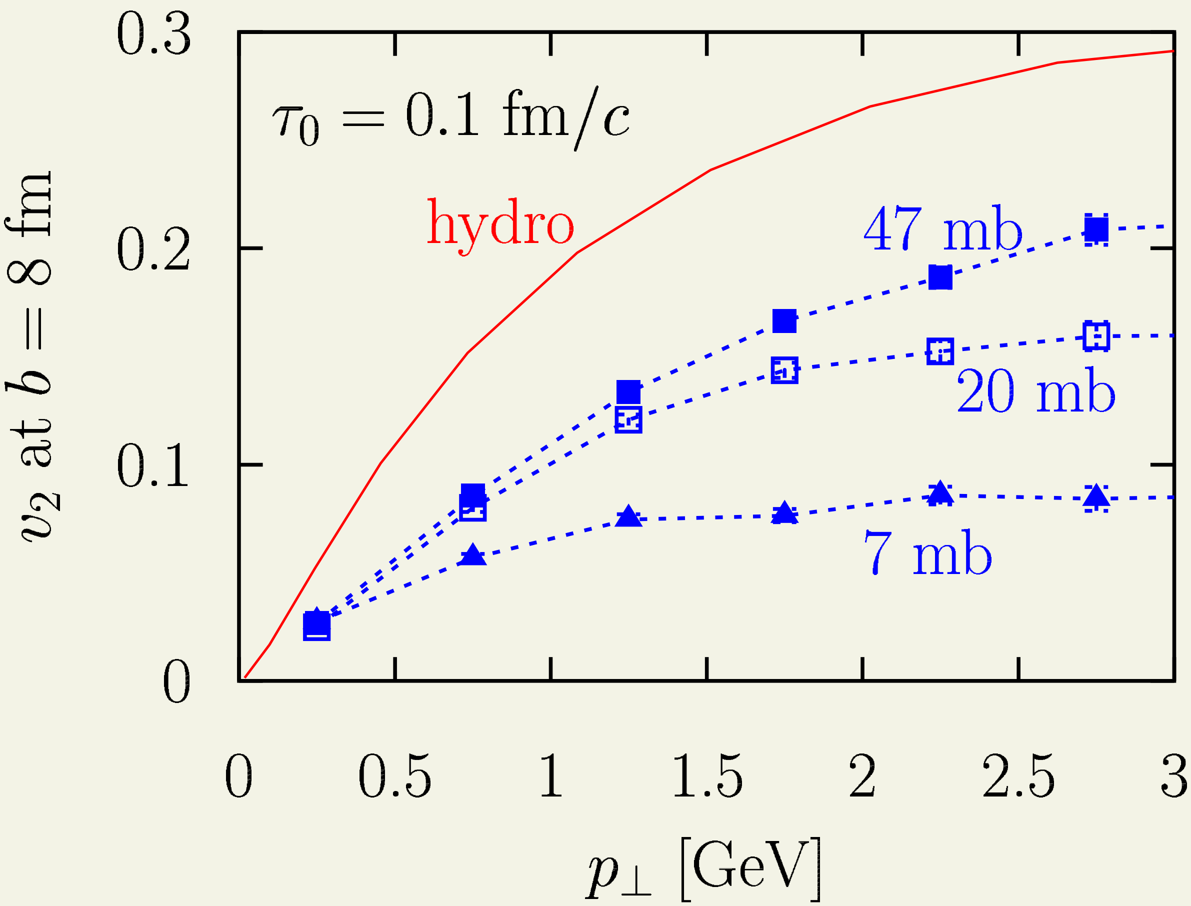
inelastic $3 \leftrightarrow 2$ helps by factor $\sim 2 - 4$ (enhances σ_{tr}), still not enough Xu, Greiner

No, still not ideal fluid

ideal hydro vs transport comparison for ultra-relativistic $\varepsilon = 3p$ with $2 \to 2$

from identical RHIC Au+Au initconds, $b = 8$ fm, binary profile, $T_0 = 0.7$ GeV

DM ('06): **final $v_2(p_T)$**



large gradients

$\Rightarrow$ even a tiny viscosity matters

Classical transport rates get arbitrarily large as $\lambda_{MFP} \to 0$

BUT, quantum mechanics: $\Delta E \cdot \Delta t \geq \hbar/2$

+ kinetic theory: $T \cdot \lambda_{MFP} \geq \hbar/3$ Gyulassy & Danielewicz '85

$$\eta \approx 4/5 \cdot T/\sigma_{tr}$$

$$s \approx 4n$$

gives minimal viscosity: $\eta/s = \frac{\lambda_{tr} T}{5} \geq 1/15$

$\mathcal{N} = 4$ **SYM + gauge-gravity duality:** $\eta/s \geq 1/4\pi$

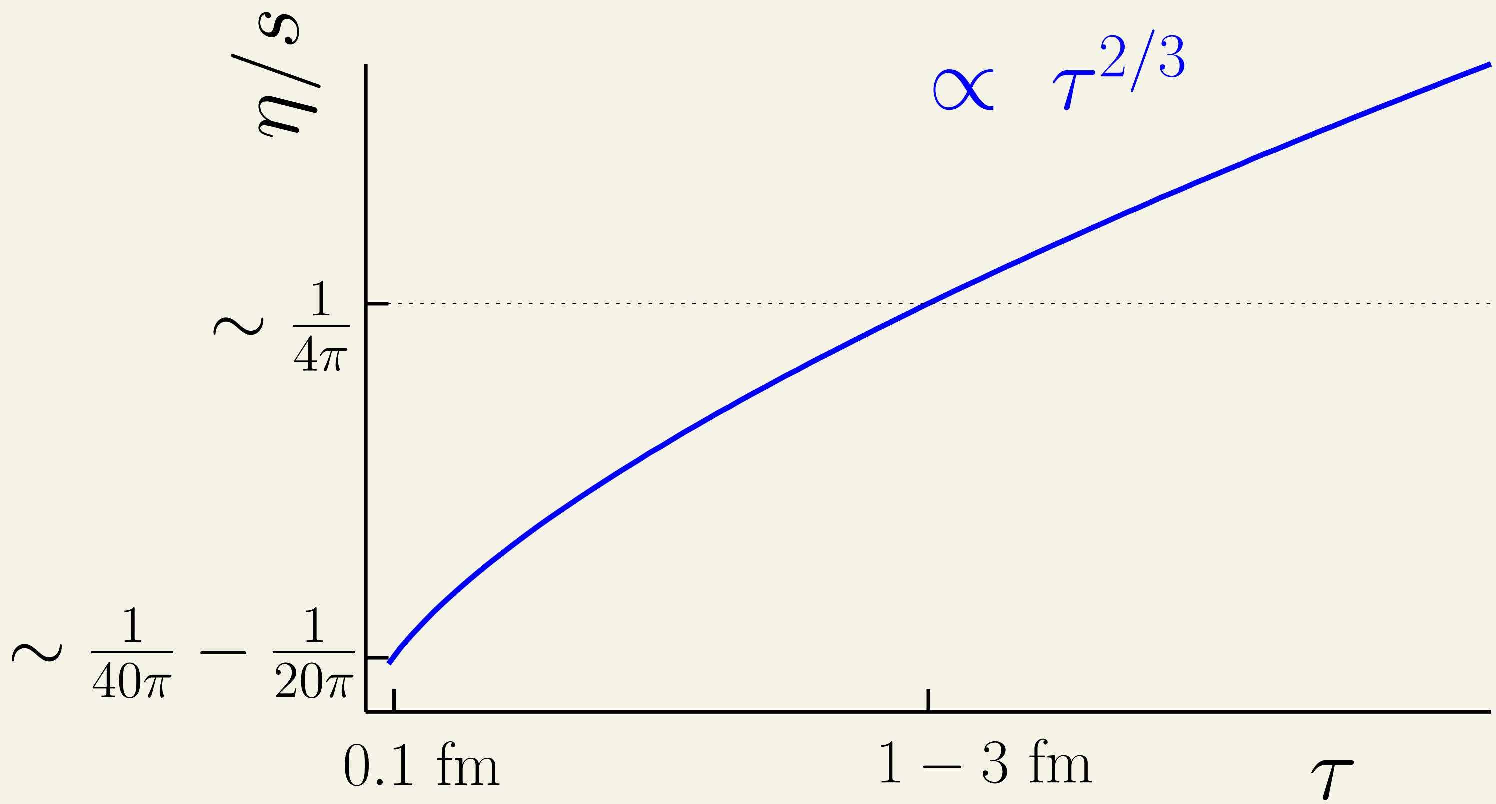
Policastro, Son, Starinets, PRL87 ('02) Kovtun, Son, Starinets, PRL94 ('05)

might be a universal lower bound - but general proof lacking

$\Rightarrow$ **no ideal fluids - "most perfect" are those with minimal viscosity**

["most" is crucial - perfect $\equiv$ ideal already since '50s]

$\sigma \approx 47$ **mb dynamics corresponds to** $\eta/s \sim \lambda_{tr} T \sim 1/(\sigma T^2)$



initially "better than perfect", after $\tau \sim 1-3$ fm "less than perfect"

$\Rightarrow \eta/s = const$ **needs growing** $\sigma(\tau) \propto 1/T^2 \propto \tau^{2/3}$

η/s for transport

"minimal" viscosity - corresponds to $\lambda_{tr} \approx 1/(3T_{eff}) \approx 0.1$ fm at $\tau_0 = 0.1$ fm

estimate from average density: $\lambda_{tr} = \frac{1}{\langle n \rangle \sigma_{tr}}$

for $b = 8$ fm @ RHIC, transport with 47 mb gives

$$\lambda_{tr}(\tau_0) = \frac{\tau_0 A_T}{\sigma_{tr} dN/d\eta} \sim 1 - 2 \times 10^{-2} \text{ fm}$$

estimate from transport opacity χ: assuming 1D Bjorken expansion

$$\chi = \int dz\, \rho(z) \sigma_{tr} \sim \int d\tau \rho_0 \frac{\tau_0}{\tau} \sigma_{tr} = \frac{\tau_0}{\lambda_{tr}(\tau_0)} \ln \frac{L}{\tau_0}$$

for $b = 8$ fm @ RHIC, transport with 47 mb gives $\chi \approx 20$

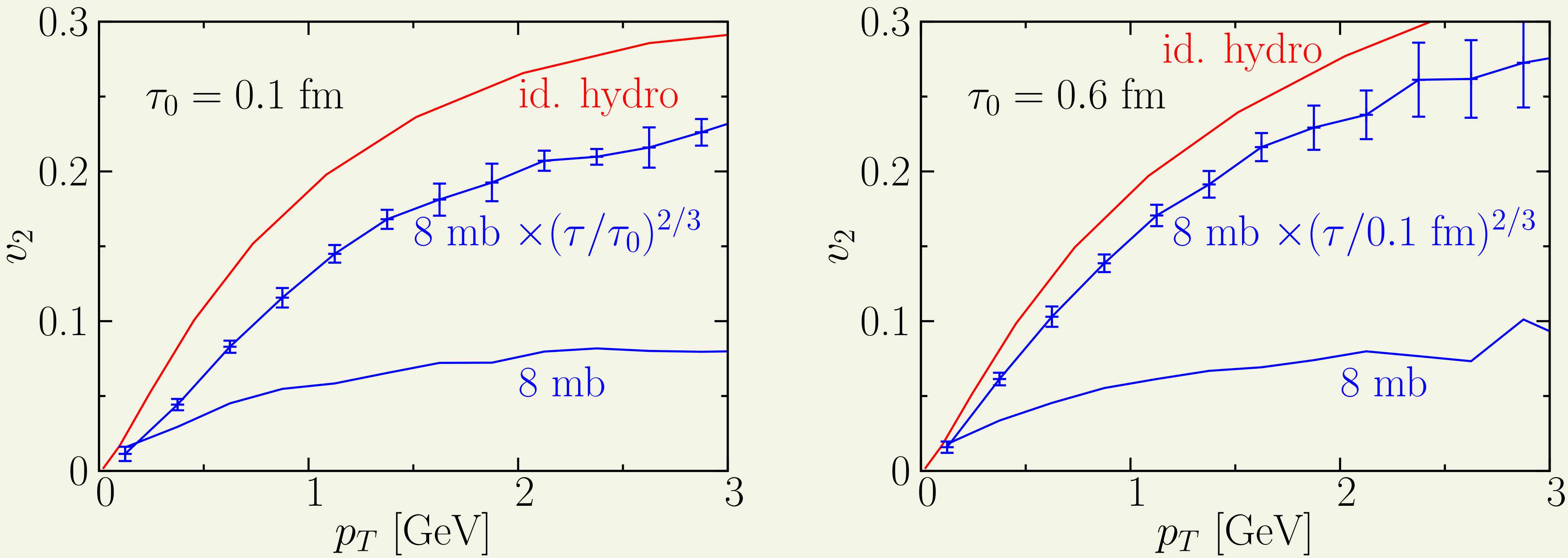
$\rightarrow \lambda_{tr}(\tau_0) \sim 1.5 - 2 \times 10^{-2}$ **fm (!)**

$\Rightarrow \sigma_{gg} \approx 50$ mb is already better than best-case scenario

redo RHIC comparison with "minimal viscosity"

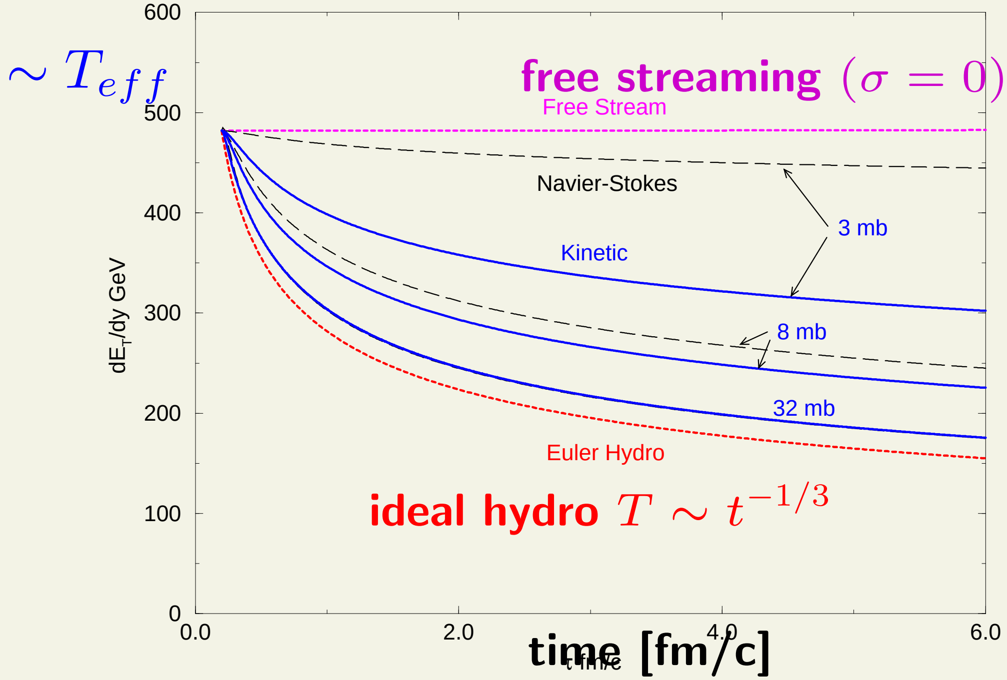
$\Rightarrow \sigma_{gg}(\tau = 0.1\ \text{fm}) \sim 4 - 9$ **mb** [4 mb for center of collision zone]

DM '06: $b = 8$ **fm**



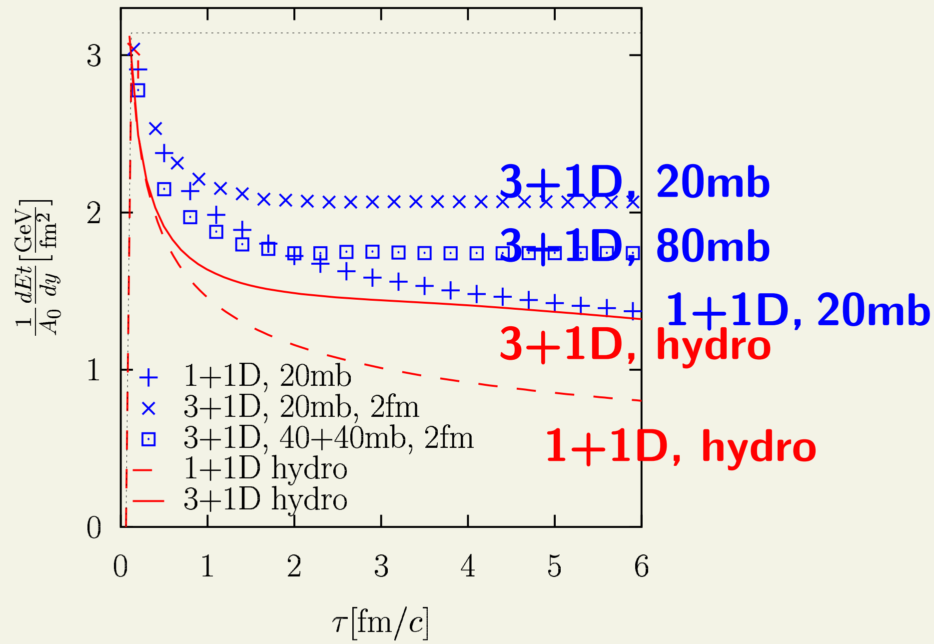
$\Rightarrow$ **still $20-30$% drop in v_2 due to dissipation, even at low p_T**

Gyulassy, Pang, Zhang ('97): 1+1D kinetic

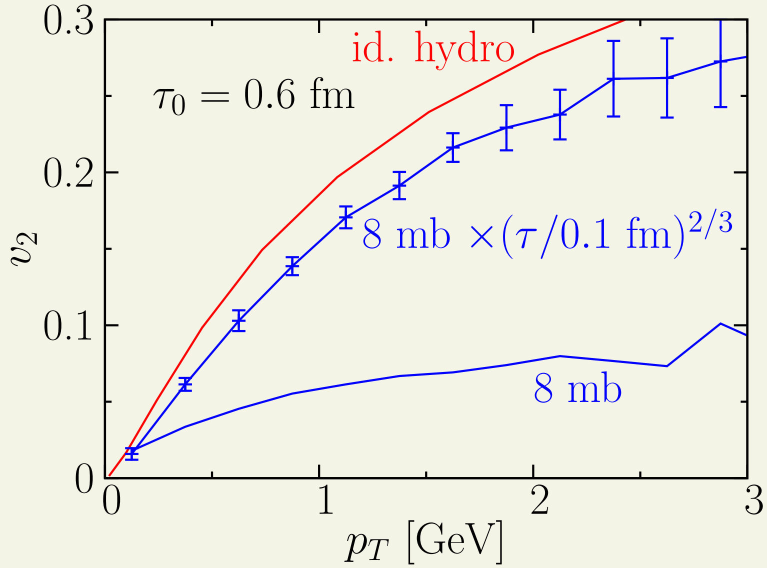


DM & Gyulassy ('00): 3+1D kinetic theory

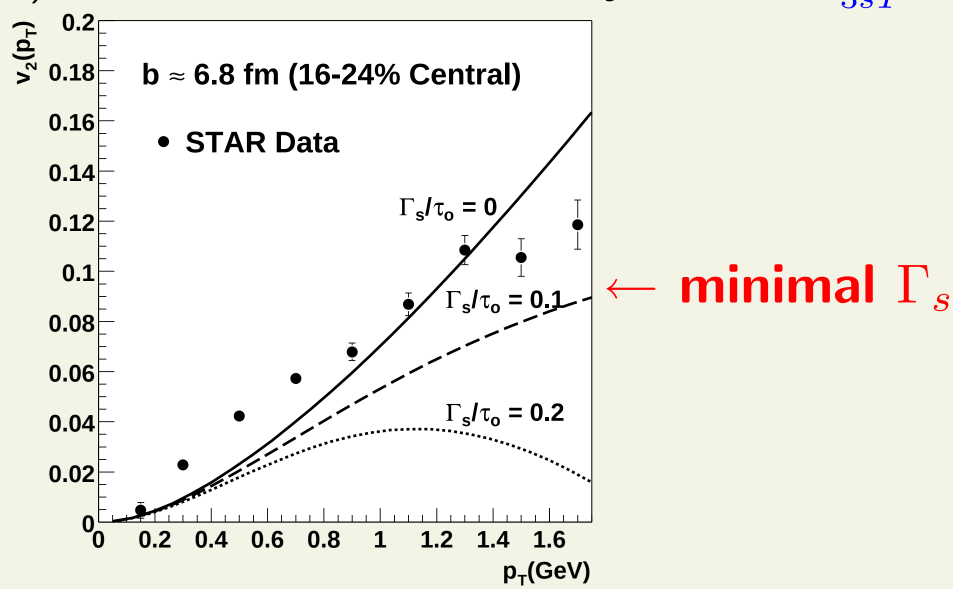
MPC vs hydro (1+1D and 3+1D)



DM ('06): $\eta/s \sim 1/(4\pi)$



Teaney ('04): **estimate at FO from id.hydro** $\Gamma_s \equiv \frac{4\eta}{3sT}$



Now go to LHC ...

and predict $v_2(p_T)$ for "minimum viscosity" system, i.e., maximal scattering rates

from a transport perspective, there are 3 relevant scales:

$$\sigma_{tr} \cdot dN/d\eta, \quad T_{eff}, \quad \text{and} \quad \tau_0/R$$

[DM & Gyulassy, NPA697 ('01)]

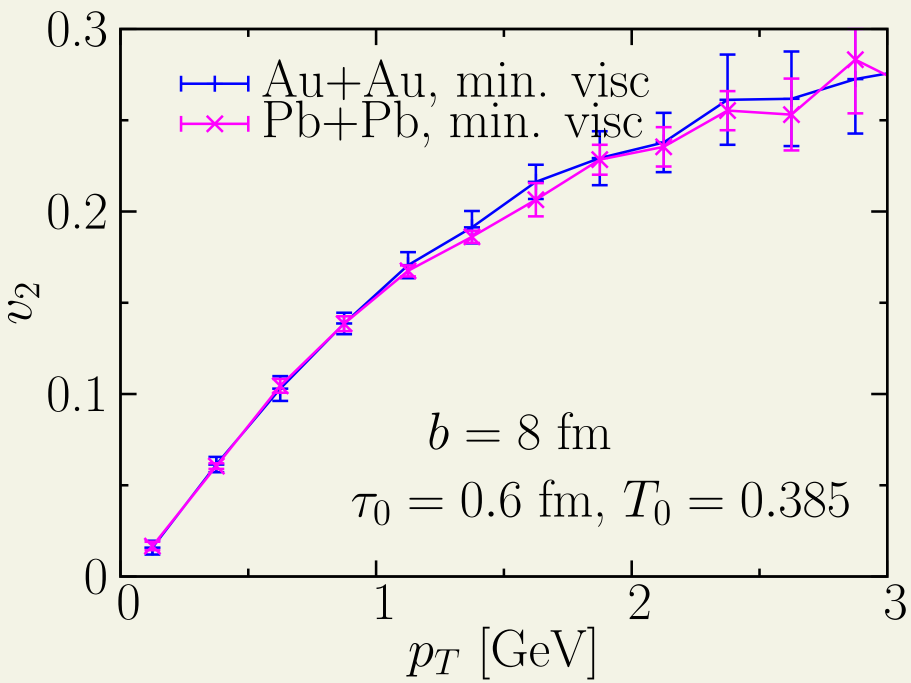
RHIC vs LHC

I. gold $\rightarrow$ lead: negligible - density profiles, $T_A(b)$, almost identical

II. larger $dN_{ch}/d\eta \sim 1200 - 2500$, highly uncertain

- **irrelevant(!)** - **transport results depend on $\lambda_{tr} \propto \sigma_{tr} \cdot dN/d\eta$, and that is fixed by the minimal viscosity requirement**

  can scale results up to any $dN/d\eta$ (with σ_{tr} reduction in inverse proportion), and v_2 stays same (ratio) DM & Gyulassy, NPA 697 ('02) -



Au+Au, $dN/d\eta$=1000, σ=8 **mb**

$\Updownarrow$

Pb+Pb, $dN/d\eta$=3000, σ=2.7 **mb**

III. higher typical momenta

- **for massless dynamics, momenta scale with initial (effective) temperature** T_{eff} **(**$\langle p_T \rangle$**, or for saturation model** Q_{sat} **)**

  provided there are no other scales in the problem

$\Rightarrow$ **universal** $v_2(\frac{p_T}{Q_s})$**, i.e.,**

$$v_2^{LHC}(p_T) \approx v_2^{RHIC}(p_T \frac{Q_s^{RHIC}}{Q_s^{LHC}})$$

[simplest example: uniform initial temperature profile]

estimate Q_s^{RHIC}/Q_s^{LHC} from saturation condition

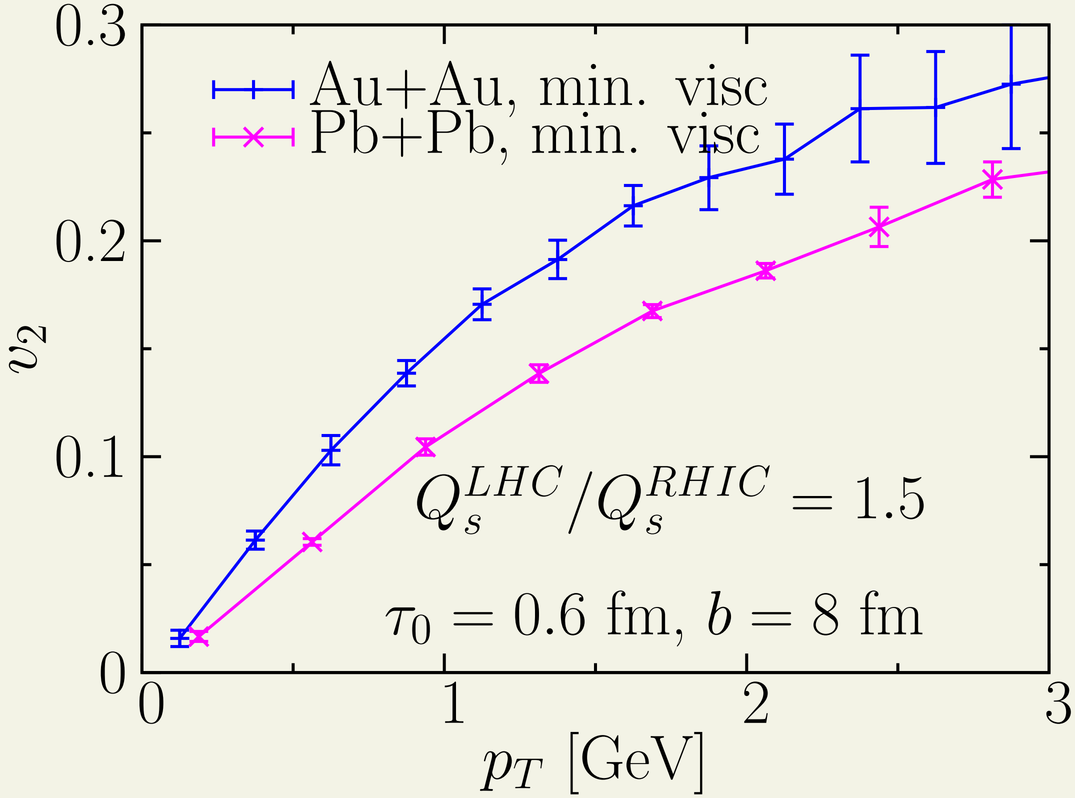
$$Q_s^2 = \frac{2\pi^2}{C_F}\alpha_S(Q_s^2)\ xG(x = \frac{Q_s}{\sqrt{s}}, Q_s^2)\ T_A$$

$\Rightarrow Q_s^{LHC}/Q_s^{RHIC} \approx 1.5$ (central collisions)

refine for $b \neq 0$ with $\langle p_T^2 \rangle$ from k_T-factorized GLR as in Adil et al, PRD73 ('06)

$$\frac{dN_g}{d^2x_T dp_T d\eta} = \frac{4\pi}{C_F}\frac{\alpha_s(p_T^2)}{p_T}\int d^2k_T\,\phi_A(x_1,\vec{p}_1,\vec{x}_T)\,\phi_B(x_2,\vec{p}_2,\vec{x}_T)$$

$$\Rightarrow Q_s^{LHC}/Q_s^{RHIC} \sim \sqrt{\frac{\langle p_T^2\rangle^{LHC}}{\langle p_T^2\rangle^{RHIC}}} \approx\ 1.3 - 1.5 \qquad \textbf{for } b = 8 \textbf{ fm}$$



from naive $v_2^{LHC}(p_T) \approx v_2^{RHIC}(p_T \frac{Q_s^{RHIC}}{Q_s^{LHC}})$

IV. but higher T_{eff} also means higher σ, since $\lambda_{tr} \approx \frac{1}{3T_{eff}}$ quantum bound

$\Rightarrow$ **need $v_2(p_T)$ for $1.3 - 1.5\times$ larger σ**

or, in other words the scaling gets modified as

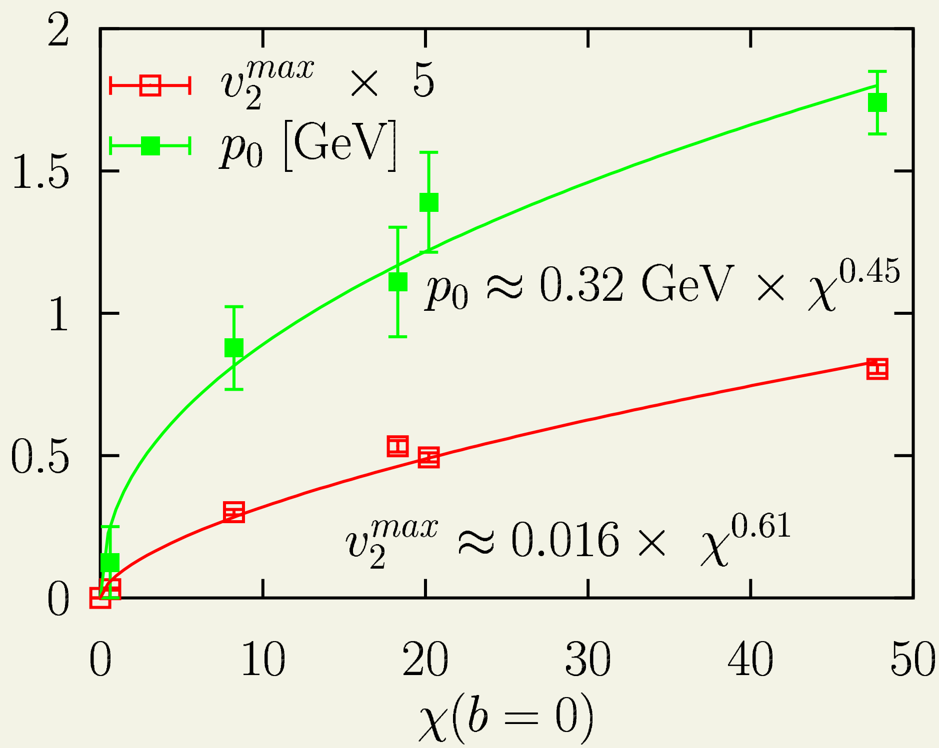
$$v_2(p_T, \frac{\eta}{s}, T_0^{eff}) = v_2(k \cdot p_T, k \cdot \frac{\eta}{s}, k \cdot T_0^{eff})$$

look for general scaling laws using the parameterization

$$v_2(p_T, \sigma) = v_2^{\max}(\sigma) \tanh \frac{p_T}{p_0(\sigma)}$$

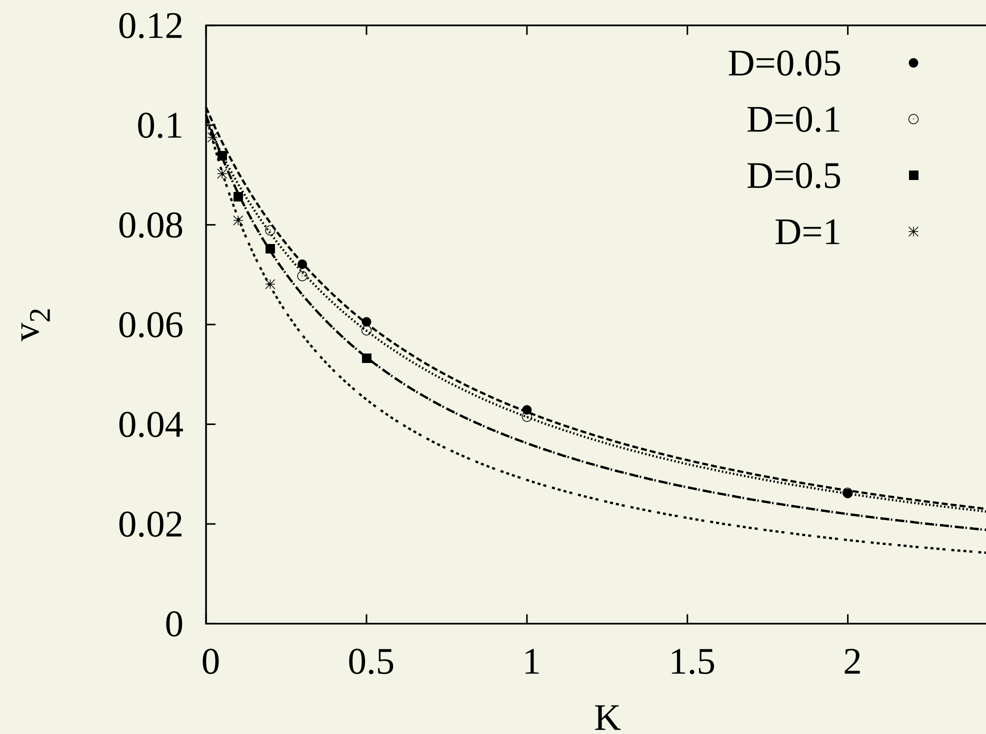
DM, JPG31 ('03): **power law fits**

$v_2^{max} \sim \sigma^\alpha$, $p_0 \sim \sigma^\beta$ $\quad (\chi \sim \sigma)$



Ollitrault & Gombeaud ('07): **rational function**

$v_2^{integrated} \sim \frac{\tilde{v}_2}{\sigma_0/\sigma+1}$ $\quad (K \sim 1/\sigma)$

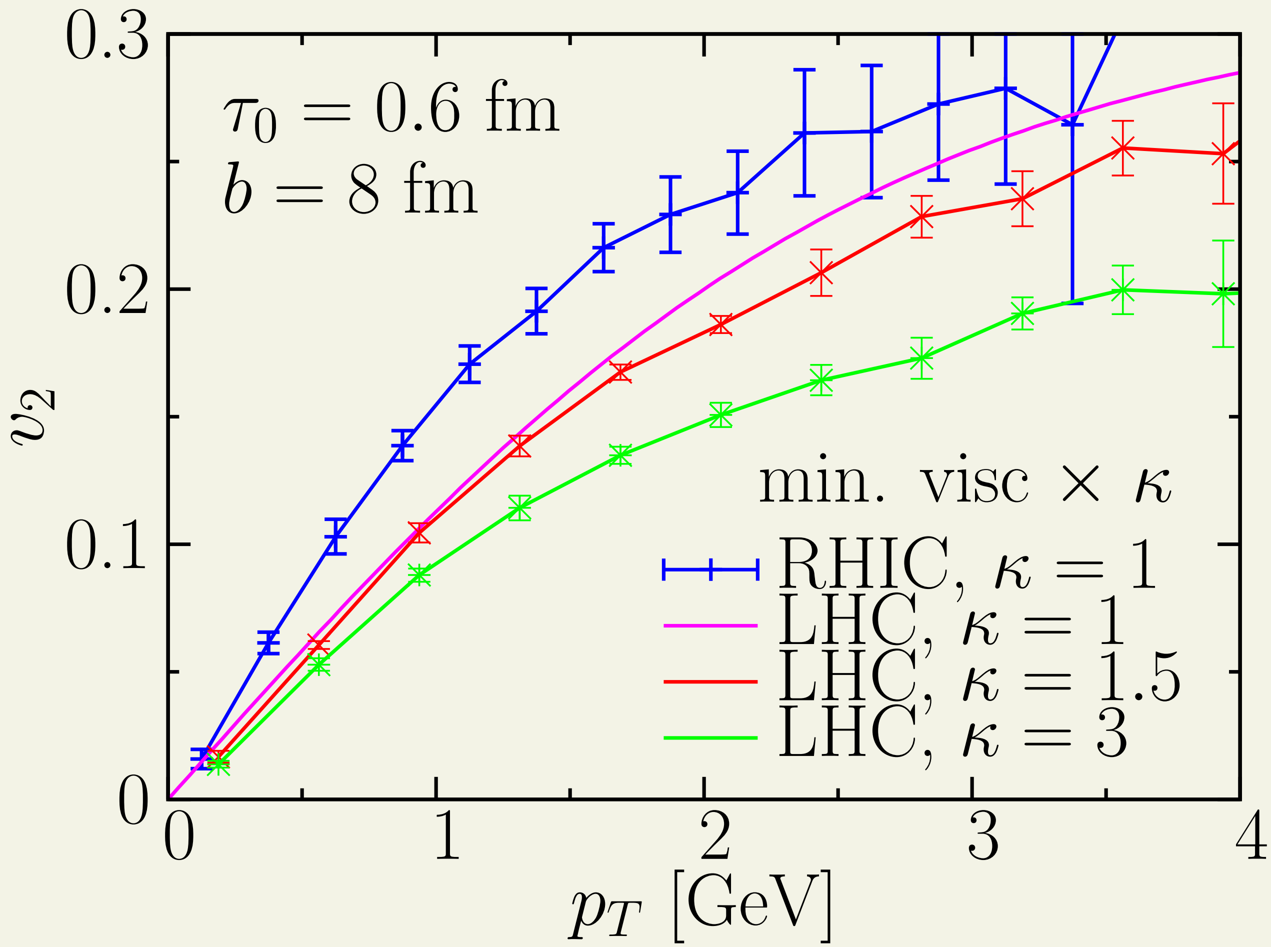


generalize rational function fits to $v_2(p_T, \sigma)$:

$v_2^{max}(\chi) = \frac{v_2^{max}}{\sigma_v/\sigma+1}$, $p_0(\chi) = \frac{p_0^{max}}{\sigma_p/\sigma+1}$ $\rightarrow$ **also give good fits** simulations for Pb+Pb with $dN_{had}(b=0)/d\eta = 3000$, binary profile

naively, "min. viscosity" $\Leftrightarrow \sigma \sim 1.3$ mb, but instead need 1.3×1.5 mb

$\Rightarrow$ obtain answer using fit functions to $v_2(p_T, \sigma)$



$$v_2 = v_2^{\max}(\sigma) \tanh \frac{p_T}{p_0(\sigma)}$$

fit results ($b = 8$ fm):

$$v_2^{max}(\sigma) \approx \frac{0.404}{0.554\, mb/\sigma + 1}$$

$$p_0(\sigma) \approx \frac{2.92\, GeV}{0.187\, mb/\sigma + 1}$$

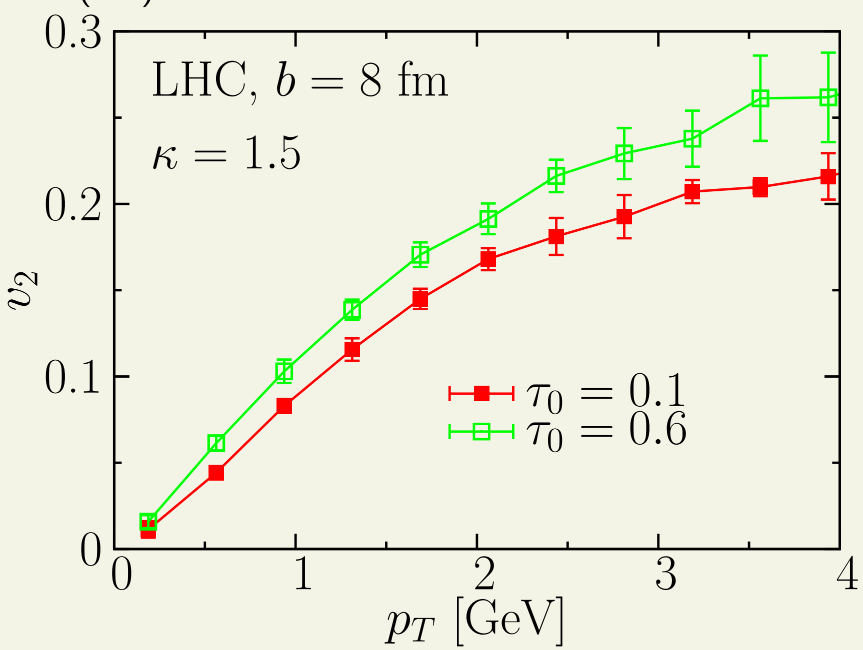
small $5-10$% increase in $v_2(p_T)$ relative to naive scaling

V. higher Q_{set} also (likely) means faster thermalization

at least from dimensions expect $\tau_0 \sim 1/Q_s$

this involves the last remaining scale τ_0/R, which controls the interplay between longitudinal and transverse dynamics

DM ('06):



needs to be studied in more detail, but promising that factor 6 decrease in τ_0 gives only about 20% decrease in v_2

$\Rightarrow$ 50% variation in τ_0 should not be too important $(< few\%)$

Conclusions / prediction

based on $2 \to 2$ covariant transport expect the following scaling for charged hadron elliptic flow to hold, perhaps within 10%, in the low-pT region up to $p_T \sim 3$ GeV at midrapidity:

$$v_2^{LHC,5500}(p_T) \approx v_2^{RHIC,200}(p_T \cdot k)$$

where $k = \frac{Q_s^{RHIC}}{Q_s^{LHC}} \approx 1.5$ is the ratio of typical initial parton momenta at $b = 8$ fm estimated from the GLR approach.

The calculation assumed that the systems formed at RHIC and the LHC both have the SAME, "minimal", shear viscosity/entropy density ratio, i.e., that scattering rates exhaust their quantum bounds. Under these conditions there are already ~ 25% dissipative corrections to elliptic flow at the LHC. It is likely that the scaling extends out to $\eta/s \sim few\ times$ the minimal value (needs more extensive testing).

Studying the evolution of other observables, such as spectra, in the parameter space of this model will allow for more robust predictions (e.g., what if k significantly differs from 1.5, precise dependence on τ_0, etc...)

Open issues

initial geometry (eccentricity)

- gluon saturation models can give $\sim 1.3\times$ larger spatial eccentricity ε than even binary profile (depends on model details)

because $v_2 \sim \varepsilon$, this can reduce cross sections but is not very likely to affect the conclusions because energy dependence of eccentricity is rather weak (only the interpretation $\to \eta/s$ changes somewhat)

missing $3 \leftrightarrow 2$ processes

for minimal viscosity, this is probably not a big issue, as the viscosity is FIXED here by the entropy. Adding extra scattering channels would decrease η below the quantum bound, unless all cross sections are reduced at the same time.