

Baryon Number and Strangeness Fluctuations

- 1) Introduction
 - QCD phase diagram, chiral limit, universality
- 2) Taylor expansion at non-vanishing B , S , Q chemical potential
 - \Rightarrow generalized susceptibilities at $\mu_{B,S,Q} = 0$
- 3) Baryon number and strangeness fluctuations at $\mu = 0$
 - preliminary results from (2+1)-flavor QCD simulations with almost realistic quark masses (RBC-Bielefeld collaboration)
- 4) Conclusions

Message: Interesting signatures for an (almost) chiral transition at $\mu_{B,S,Q} = 0$ may show up in higher moments

Baryon Number and Strangeness Fluctuations

RBC-Bielefeld collaboration

M. Cheng^a, N. H. Christ^a, J. van der Heide^c, C. Jung^b, F. Karsch^{b,c}, O. Kaczmarek^c,
E. Laermann^c, R. D. Mawhinney^a, C. Miao^c, P. Petreczky^{b,d}, K. Petrov^e, C. Schmidt^b
and T. Umeda^f

^a Physics Department, Columbia University, New York, NY 10027, USA

^b Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

^c Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany

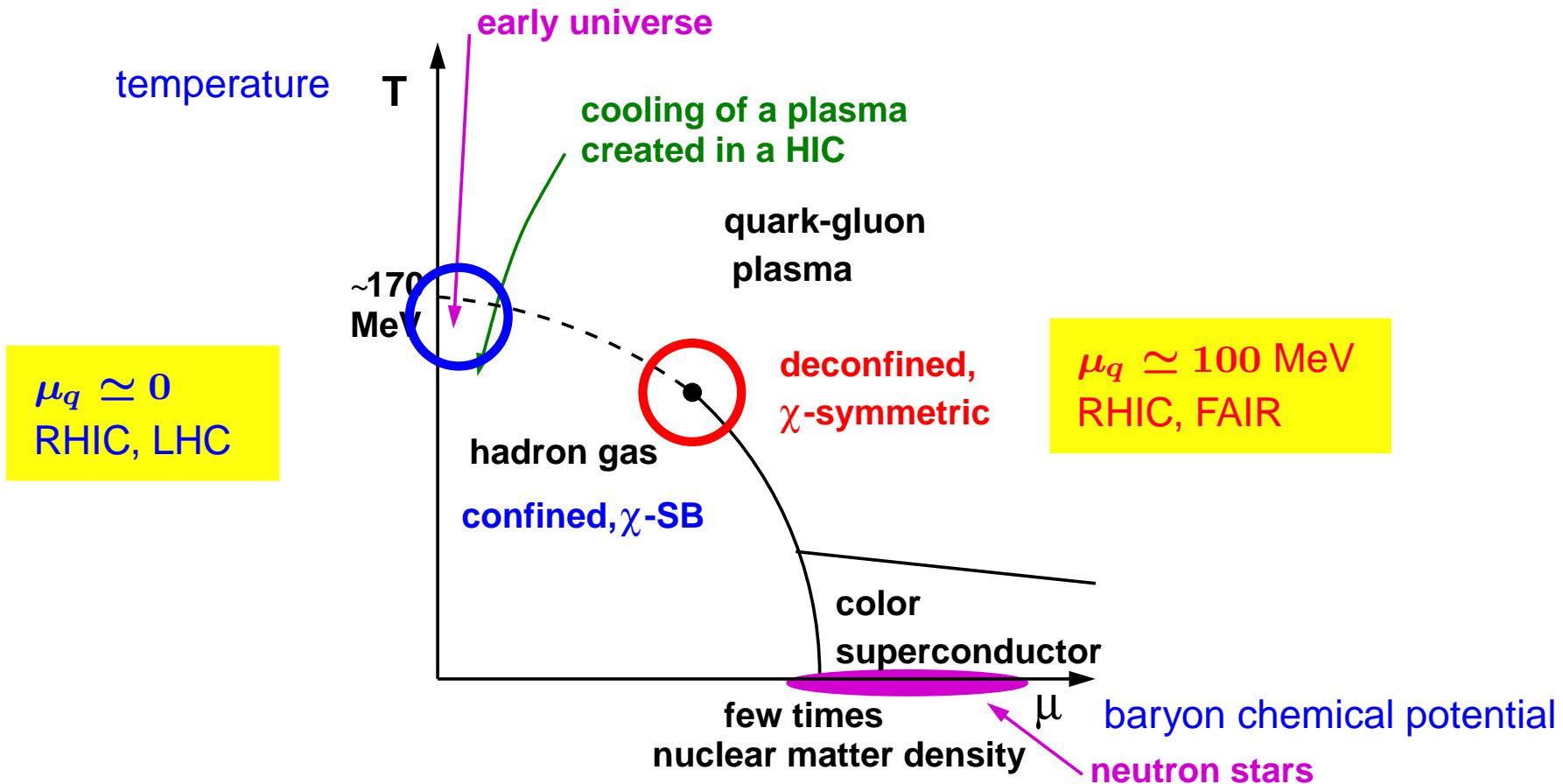
^d RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

^e Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen, Denmark

^f Physics Department, Tsukuba University, Tsukuba, Japan

Phase diagram of strongly interacting matter

RHIC I/II & LHC \Leftrightarrow LGT at vanishing chemical potential
FAIR@GSI & RHIC at low energy \Leftrightarrow LGT at non zero chemical potential

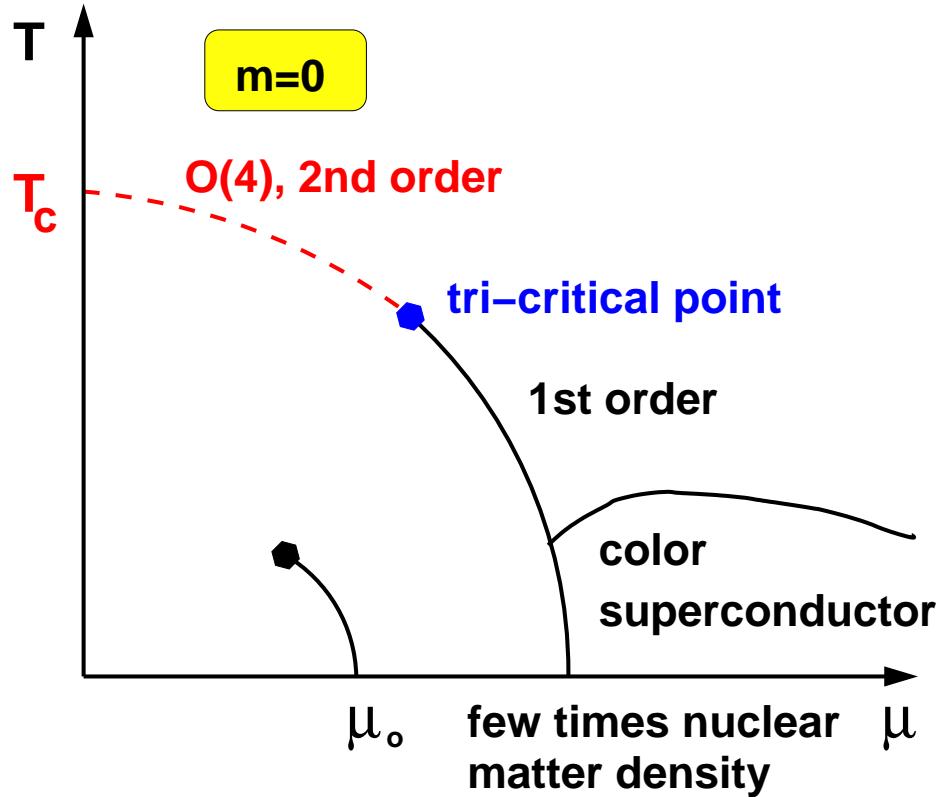
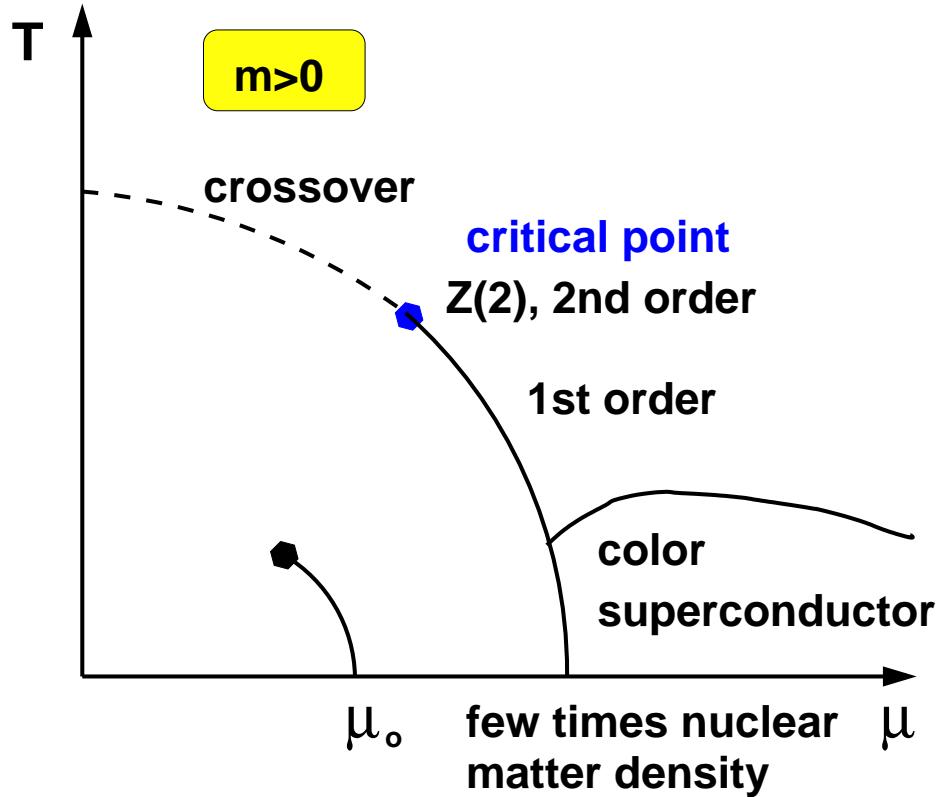


Generic QCD phase diagram

QCD, physical quark masses

QCD, chiral limit (u, d quarks)

$$\mu \equiv \mu_B \geq 0, \mu_s = \mu_Q = 0$$



Hadronic fluctuations and chiral symmetry restoration

- $\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q, \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q, \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q + \mu_S$
- expect 2^{nd} order transition in 3-d, O(4) symmetry class;

$$\text{scaling field: } t = \left| \frac{T - T_c}{T_c} \right| + A \left(\left(\frac{\mu_u}{T_c} \right)^2 + \left(\frac{\mu_d}{T_c} \right)^2 \right) + B \frac{\mu_u}{T_c} \frac{\mu_d}{T_c}$$

$$\text{singular part: } f_s(T, \mu_u, \mu_d) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{2-\alpha}$$

$$\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_{u,d}^2} \sim t^{1-\alpha} \quad , \quad \frac{\partial^4 \ln \mathcal{Z}}{\partial \mu_{u,d}^4} \sim t^{-\alpha} \quad (\mu = 0)$$

- O(4)/O(2): $\alpha < 0$, small \Rightarrow
 - $\langle (\delta N_{u,d})^2 \rangle$ dominated by T-dependence of regular part
 - $\langle (\delta N_{u,d})^4 \rangle$ develops a cusp
- How does μ_S couple to the singular part of the light quark sector?

Hadronic fluctuations and chiral symmetry restoration

- expect 2^{nd} order transition in 3-d, O(4) symmetry class;

$$\text{scaling field: } t = \left| \frac{T - T_c}{T_c} \right| + A \left(\left(\frac{\mu_q}{T_c} \right)^2 - \left(\frac{\mu_{crit}}{T_c} \right)^2 \right), \quad \mu_{Q,S} = 0$$

$$\text{singular part: } f_s(T, \mu_q) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{2-\alpha}$$

$$\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_q^2} \sim t^{-\alpha} \quad , \quad \frac{\partial^4 \ln \mathcal{Z}}{\partial \mu_q^4} \sim t^{-(2+\alpha)} \quad (\mu_{\equiv} \mu_{u,d} > 0)$$

- O(4)/O(2): $\alpha < 0$, small \Rightarrow

$\langle (\delta N_q)^2 \rangle$ develops a cusp

$\langle (\delta N_q)^4 \rangle$ diverges on the O(4) critical line;

strength $\sim \left(\frac{\mu_{crit}}{T_c} \right)^4$ ($\sim 10^{-4}$ at RHIC and LHC)

Hadronic fluctuations at $\mu = 0$ from Taylor expansion coefficients for $\mu > 0$

$n_f = 2 + 1$, $m_\pi \simeq 210$ MeV: RBC-Bielefeld, preliminary

- Taylor expansion of bulk thermodynamics in terms of $\mu_{q,s}$

$$\begin{aligned}\frac{p}{T^4} &\equiv \frac{1}{VT^3} \ln Z(V, T, \mu_q, \mu_s) \\ &= \sum_{i,j} c_{i,j} \left(\frac{\mu_q}{T}\right)^i \left(\frac{\mu_s}{T}\right)^j\end{aligned}$$

- expansion coefficients evaluated at $\mu_{q,s} = 0$ are related to fluctuations of B , S , Q at $\mu_{B,S,Q} = 0$:

↑ baryon number, strangeness, charge fluctuations

event-by-event fluctuations at RHIC and LHC

Hadronic fluctuations at $\mu = 0$ from Taylor expansion coefficients for $\mu > 0$

$n_f = 2 + 1$, $m_\pi \simeq 210$ MeV: RBC-Bielefeld, preliminary

- higher derivatives \Rightarrow higher moments
- mixed derivatives \Rightarrow correlations

$$2c_2^x = \frac{\partial^2 p/T^4}{\partial(\mu_x/T)^2} = \frac{1}{VT^3} \langle (\delta N_x)^2 \rangle_{\mu=0} = \frac{1}{VT^3} \langle N_x^2 \rangle_{\mu=0}$$

$$24c_4^x = \frac{\partial^4 p/T^4}{\partial(\mu_x/T)^4} = \frac{1}{VT^3} (\langle (\delta N_x)^4 \rangle - 3 \langle (\delta N_x)^2 \rangle)_{\mu=0} = \frac{1}{VT^3} (\langle N_x^4 \rangle - 3 \langle N_x^2 \rangle)_{\mu=0}$$

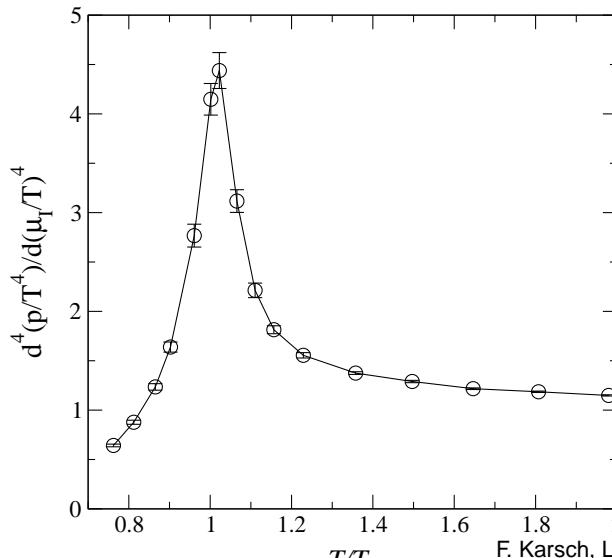
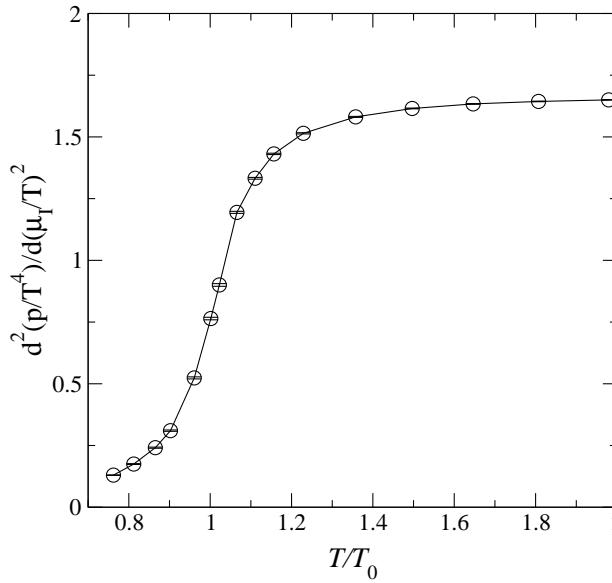
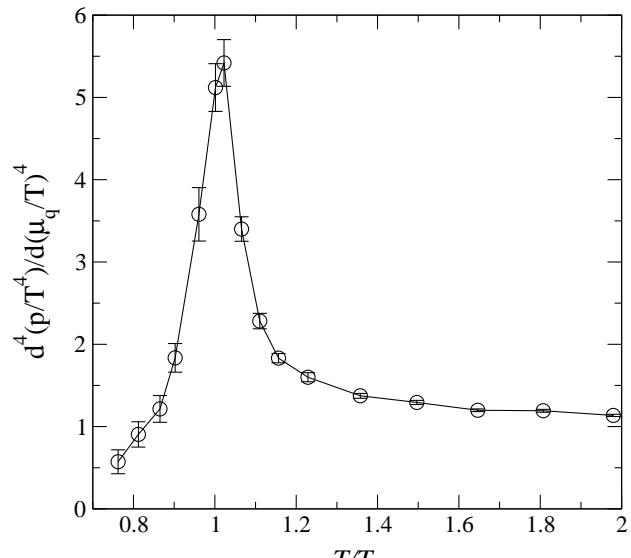
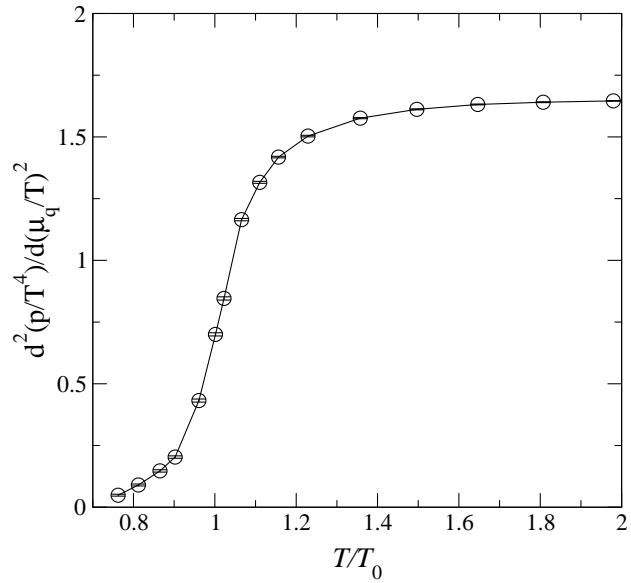
$$4c_{22}^{xy} = \frac{\partial^4 p/T^4}{\partial(\mu_x/T)^2 \partial(\mu_y/T)^2} = \frac{1}{VT^3} [\langle N_x^2 N_y^2 \rangle - 2 \langle N_x N_y \rangle^2 - \langle N_x^2 \rangle \langle N_y^2 \rangle]_{\mu=0}$$

with $x = q, s$

$$\Rightarrow \frac{\langle N_q^2 N_s^2 \rangle - \langle N_s^2 \rangle \langle N_q^2 \rangle}{\langle N_s^2 \rangle} = \frac{4c_{22}^{qs} + 2(c_{11}^{qs})^2}{2c_2^s}$$

Quark number and isospin fluctuations at $\mu_B = 0$ in 2-flavor QCD ($m_\pi \simeq 770 \text{ MeV}$)

C. Allton et al. (Bielefeld-Swansea), PRD71 (2005) 054508

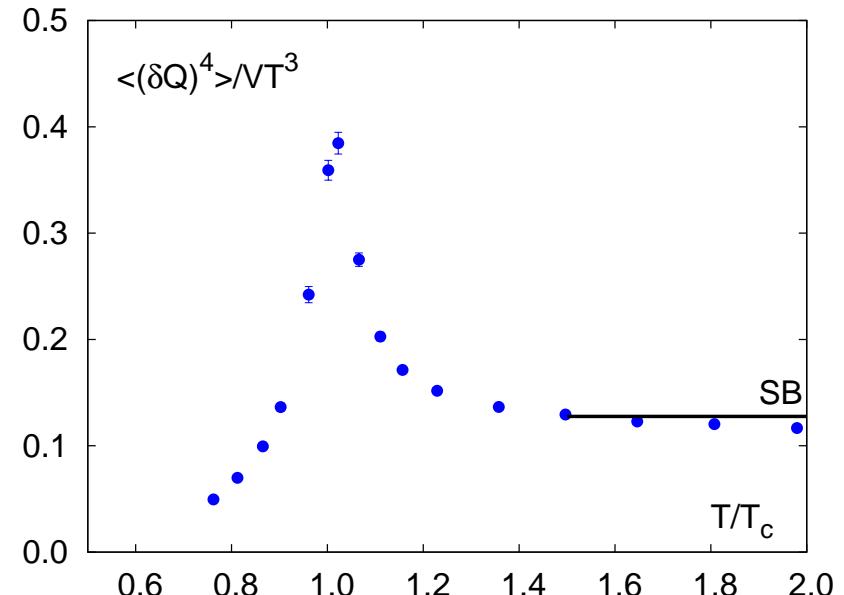
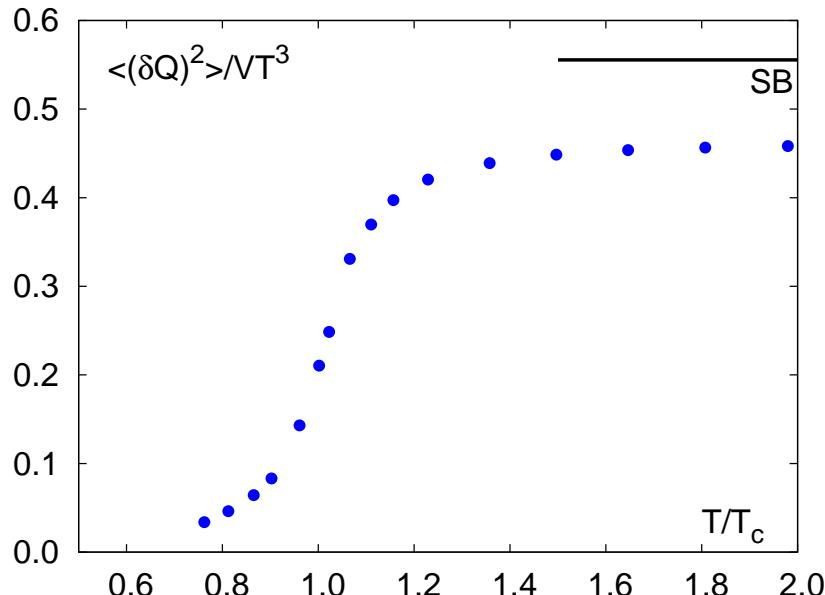


Charge fluctuations at $\mu_B = 0$

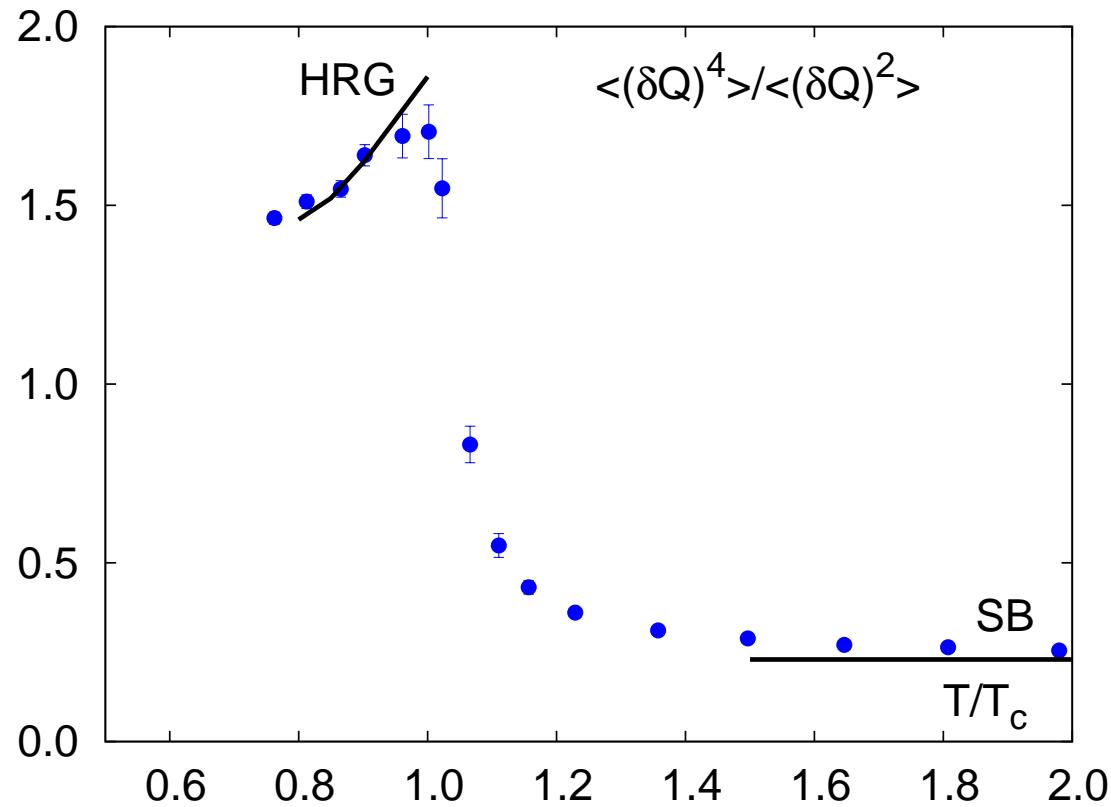
$n_f = 2, m_\pi \simeq 770$ MeV: S. Ejiri, FK, K.Redlich, PLB633 (2006) 275

- second moment: $\langle(\delta Q)^2\rangle = \langle Q^2\rangle$ increases monotonically to ideal gas value;
- fourth moment: $\langle(\delta Q)^4\rangle = \langle Q^4\rangle - 3 \langle Q^2\rangle$ develops a cusp at "phase" transition

$$\langle(\delta Q)^4\rangle = \frac{1}{1296} (17\langle(\delta N_q)^4\rangle + 65\langle(\delta N_I)^4\rangle + 54\langle(\delta N_q)^2(\delta N_I)^2\rangle) \sim t^{-\alpha}$$



Charge fluctuations at $\mu_B = 0$



Strangeness fluctuations

RBC-Bielefeld, preliminary

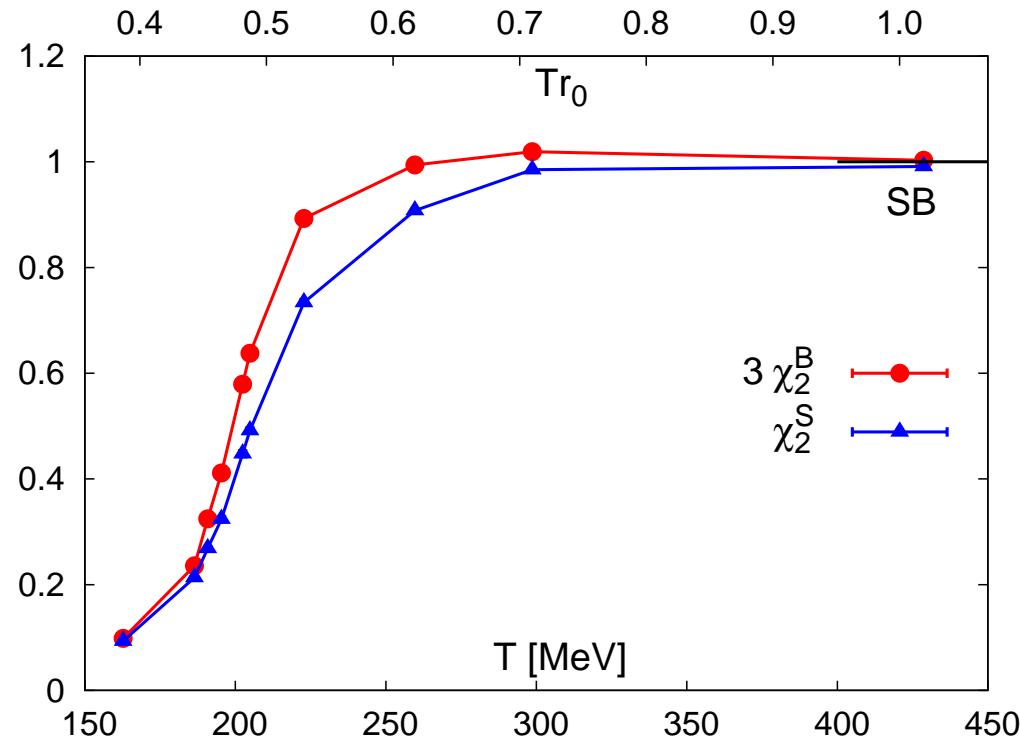
first results on strangeness fluctuations and their correlation with baryon number fluctuations in QCD with physical, dynamical strangeness sector and almost physical light quarks

- (2+1)-flavor QCD, $m_\pi \simeq 210$ MeV
- lattice sizes: $16^3 \times 4$ and $24^3 \times 6$ (today only $N_\tau = 4$)
- calculations with improved staggered fermions (p4)
⇒ small cut-off dependence in the high temperature, ideal gas limit

Quadratic fluctuations of baryon number and strangeness in (2+1)-flavor QCD

RBC-Bielefeld, preliminary

vanishing chemical potentials:



$$\chi_2^B = \frac{1}{VT^3} \langle N_B^2 \rangle$$

$$\chi_2^S = \frac{1}{VT^3} \langle N_S^2 \rangle$$

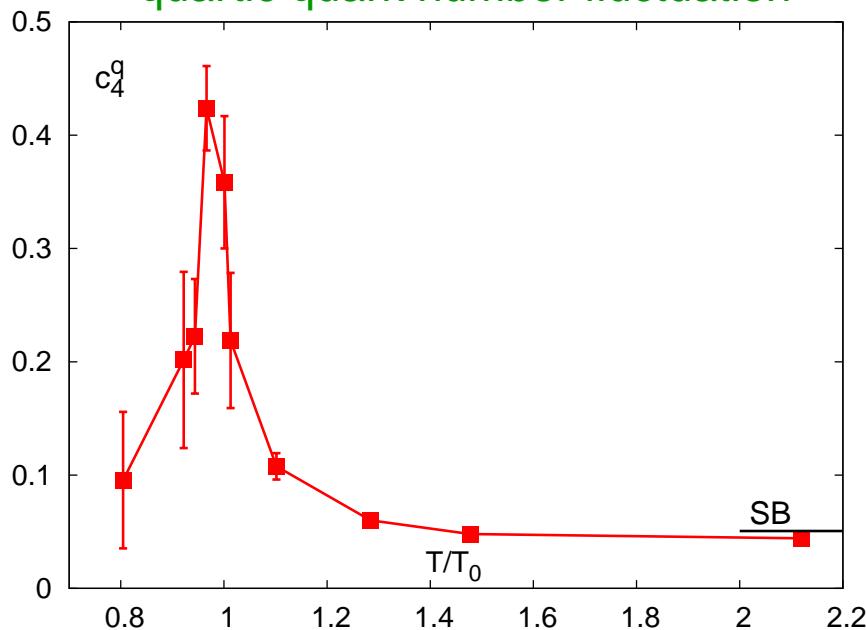
⇒ smooth change of quadratic fluctuations across transition region

chiral limit: $\chi_2^B, \chi_2^S \sim |T - T_c|^{1-\alpha} + \text{regular}$

Quartic fluctuations of baryon number and strangeness in (2+1)-flavor QCD

$$c_4^q = \frac{1}{4!} \frac{1}{VT^3} (\langle N_q^4 \rangle - 3\langle N_q^2 \rangle^2)$$

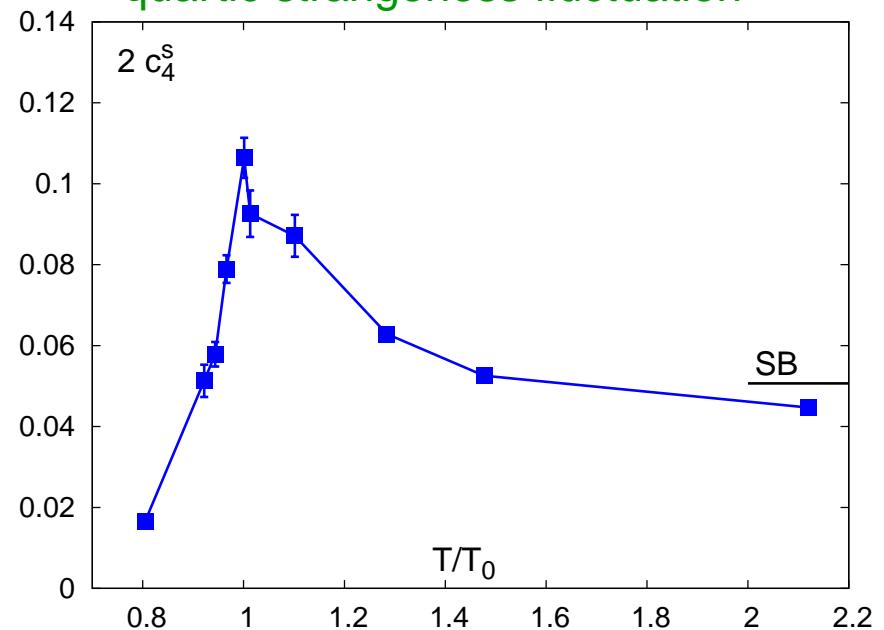
quartic quark number fluctuation



RBC-Bielefeld, preliminary

$$c_4^s = \frac{1}{4!} \frac{1}{VT^3} (\langle N_s^4 \rangle - 3\langle N_s^2 \rangle^2)$$

quartic strangeness fluctuation



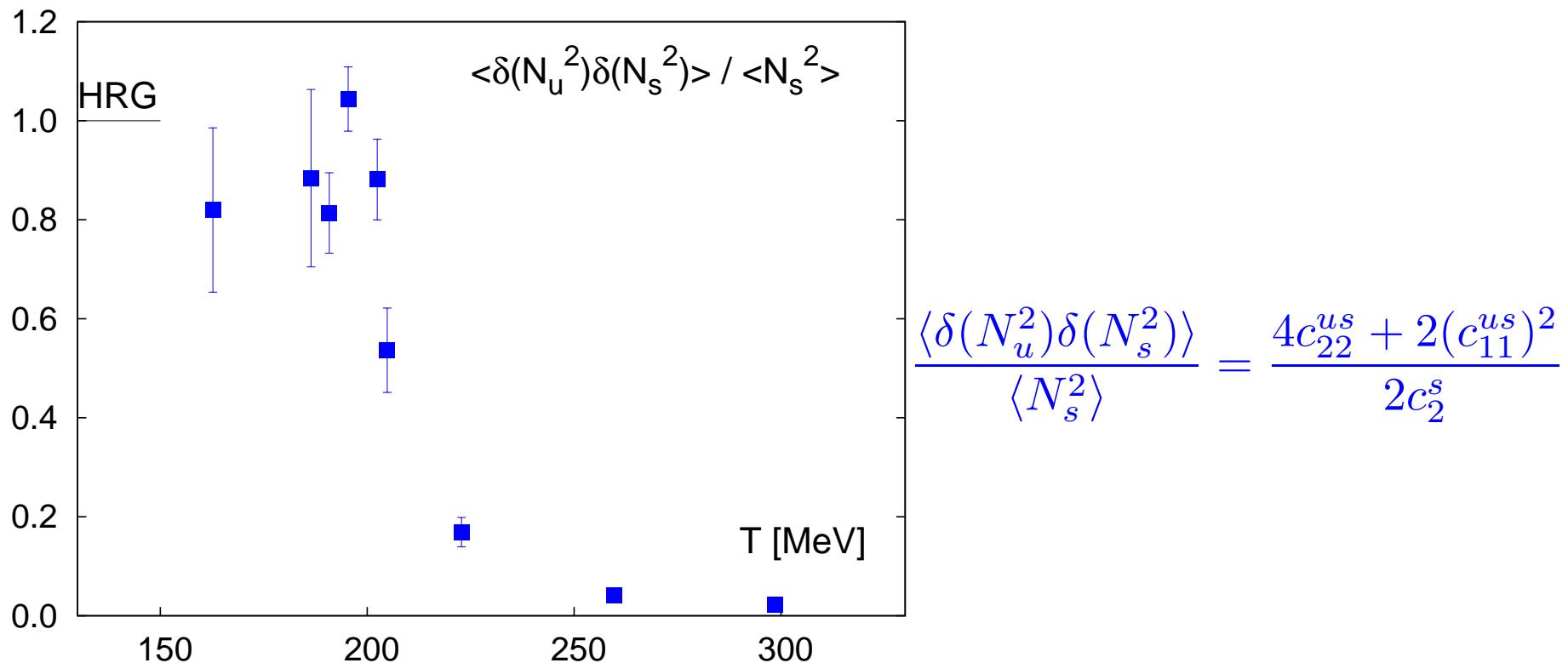
chiral limit: $c_4^q, c_4^s \sim |T - T_c|^{-\alpha} + \text{regular}$

⇒ large light quark (baryon) number fluctuations

⇒ enhanced strangeness fluctuations (factor ~ 2)

Correlation between light quark number and strangeness fluctuations

RBC-Bielefeld, preliminary

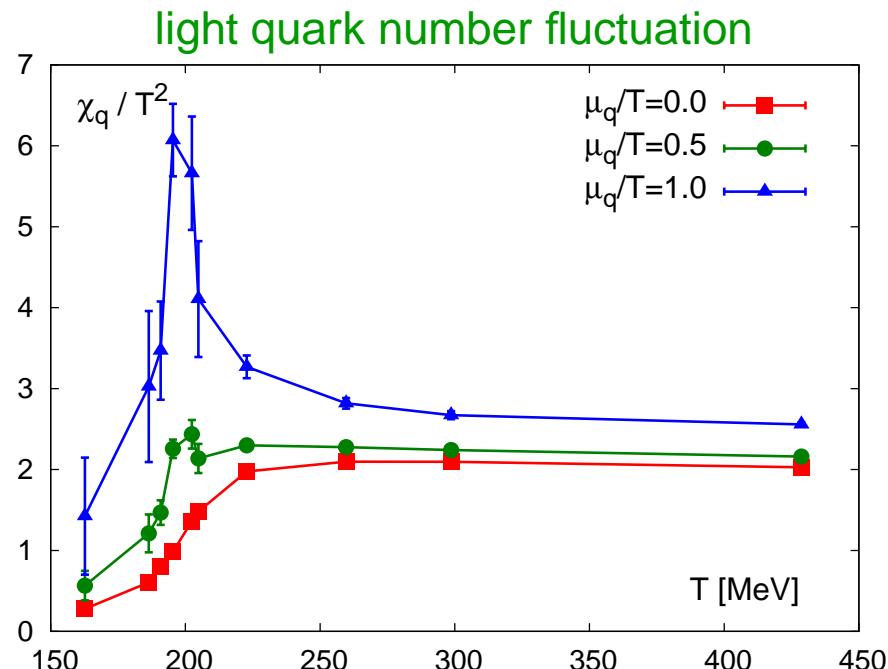


- ⇒ $T < T_c$: strangeness fluctuations coupled to light quark fluctuations
- ⇒ $T \gtrsim 1.3T_c$: strangeness and light quark fluctuations uncorrelated

$\mu_q > 0$: Fluctuations of baryon number and strangeness in (2+1)-flavor QCD

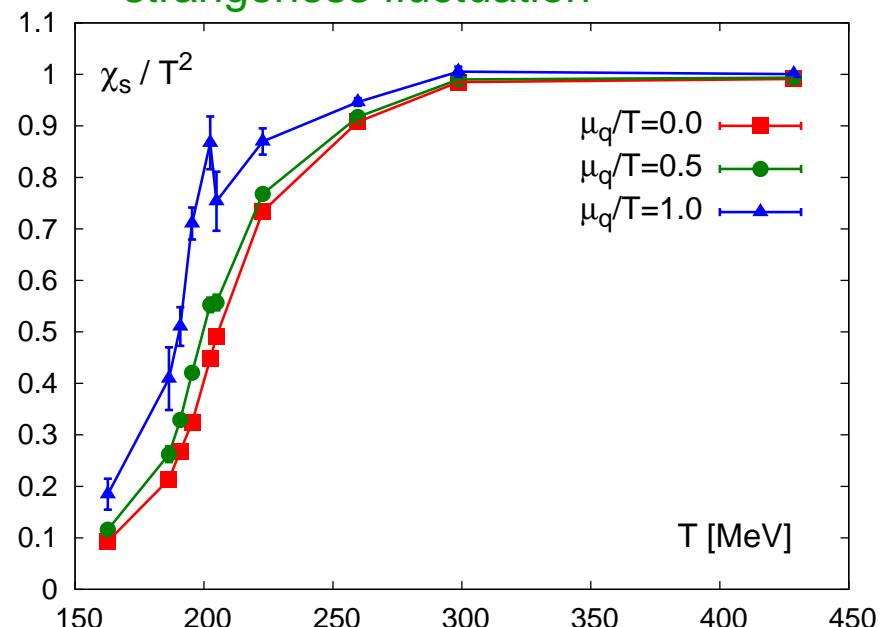
RBC-Bielefeld, preliminary

$$\chi_q/T^2 = 2c_2^q + 12c_4^q(\mu_q/T)^2$$



$$\chi_s/T^2 = 2c_2^s + 2c_{22}^{qs}(\mu_q/T)^2$$

strangeness fluctuation



⇒ large quark number fluctuations

⇒ enhanced strangeness fluctuations (factor ~ 2)

Conclusions

- Taylor expansion coefficients of finite density QCD provide information on fluctuations at $\mu_{B,Q,S} = 0$
- fourth order moments show strong signal of the "phase" transition detectable at RHIC and LHC in event-by-event fluctuations??
- rapid rise of baryon number (and charge) fluctuations with increasing baryon number density (μ_B/T);
with less strength also visible in strangeness fluctuations
- fluctuations at low T still compatible with HRG model;
deviations become visible for $T \gtrsim 0.95T_c$, e.g. in c_8^q