

# Jet quenching parameter $\hat{q}$ from Wilson loops in a thermal environment

D. Antonov, D. Dietrich, H.-J. Pirner

Institut für Theoretische Physik, Universität Heidelberg



MARIE CURIE ACTIONS

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# Introduction

A hard collision  $\Rightarrow$  an energetic parton, which traverses a finite-size  $L$  medium.

The average energy loss of the parton due to gluon radiation (R. Baier, Yu. Dokshitzer, A. Mueller, S. Peigne, D. Schiff, '96):

$$\Delta E = \frac{\alpha_s}{8} C_R \hat{q} L^2.$$

$E \propto L^2 \Rightarrow$  radiative parton energy loss  $\gg$  collisional energy loss  $\propto L$ .

The jet quenching parameter  $\hat{q}$  is the  $p_\perp^2$  transferred from the medium to the parton per distance travelled.

For a dilute plasma with the mean free path  $\lambda$ :

$$\hat{q} \sim \frac{T^2}{\lambda}.$$

# Introduction

$$\lambda \sim \frac{1}{n\sigma_t},$$

where  $n \sim T^3$  is the particle-number density,  $\sigma_t$  is the Coulomb transport cross section:

$$\sigma_t = \int d\sigma(1 - \cos \theta) \sim g^4 \int_{(gT)^2} \frac{d^2 p_\perp}{p_\perp^4} \frac{p_\perp^2}{T^2} \sim \frac{g^4}{T^2} \ln \frac{1}{g} \Rightarrow$$

$$\hat{q}_{\text{weak-coupling}} \sim \alpha_s^2 T^3 \ln \frac{1}{g}.$$

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An attempt to calculate  $\hat{q}$  at strong coupling was done in the  $\mathcal{N} = 4$  SYM by H. Liu, K. Rajagopal, and U.A. Wiedemann, '06:

$$\hat{q}_{\text{SYM}} = \frac{\pi^{3/2} \Gamma(3/4)}{\Gamma(5/4)} \sqrt{\lambda_{t'H}} T^3$$

in the large- $N_c$  and large- $\lambda_{t'H}$  limits.

# Introduction

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Differences of  $\mathcal{N} = 4$  SYM from QCD at  $T = 0$ :

- the theory is conformal, supersymmetric, has no running coupling and no confinement;
- no dynamical quarks, no chiral symmetry and its breaking;
- additional scalar and fermionic fields in the adjoint representation.

However, at finite  $T$

- supersymmetry is broken;
- in QCD above  $T_c$  there is no confinement and no chiral condensate either.

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A motivation for the present research: to calculate  $\hat{q}$  in some model of QCD, which does have conformal anomaly and confinement.

# The stochastic vacuum model

Non-Abelian Stokes theorem and the cumulant expansion yield

$$\begin{aligned} \langle W(C) \rangle \simeq & \frac{1}{\text{tr } \hat{1}} \text{tr} \exp \left[ -\frac{1}{8} \int_{\Sigma(C)} d\sigma_{\mu\nu}(x) \int_{\Sigma(C)} d\sigma_{\lambda\rho}(x') T^a T^b \times \right. \\ & \left. \times g^2 \langle F_{\mu\nu}^a(x) F_{\lambda\rho}^b(x') \rangle \right]. \end{aligned}$$

The stochastic vacuum model suggests a parametrization for the confining part of the correlator:

$$g^2 \langle F_{\mu\nu}^a(x) F_{\lambda\rho}^b(x') \rangle = \delta^{ab} \cdot \frac{g^2 \langle (F_{\mu\nu}^a)^2 \rangle}{12(N_c^2 - 1)} (\delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda}) D(x - x'),$$

where (A. Di Giacomo et al., '96, '02; G.S. Bali, N. Brambilla, A. Vairo, '97)

$$D(x) = e^{-\mu|x|}, \quad \mu = 894 \text{ MeV}.$$

# The stochastic vacuum model

For a fundamental Wilson loop whose contour  $C$  has a size  $\gg \mu^{-1}$ :

$$\sigma = \frac{\pi C_F}{12(N_c^2 - 1)} \frac{g^2 \langle (F_{\mu\nu}^a)^2 \rangle}{\mu^2}, \quad \text{where } C_F = \frac{N_c^2 - 1}{2N_c},$$

which yields for  $N_c = 3$

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At  $T > 0$ , the correlator  $\langle F_{\mu\nu}^a(x) F_{\lambda\rho}^b(x') \rangle$  splits into

$$\langle E_i^a(x) E_k^b(x') \rangle, \quad \langle B_i^a(x) B_k^b(x') \rangle, \quad \langle E_i^a(x) B_k^b(x') \rangle,$$

where  $E_i = F_{i4}$ ,  $B_i = \frac{1}{2} \varepsilon_{ijk} F_{jk}$ .

# The stochastic vacuum model

At  $T > T_c = 270 \text{ MeV}$ :

- $\langle E_i^a(x) E_k^b(x') \rangle$  vanishes (deconfinement);
- $\langle E_i^a(x) B_k^b(x') \rangle \ll \langle B_i^a(x) B_k^b(x') \rangle$ .

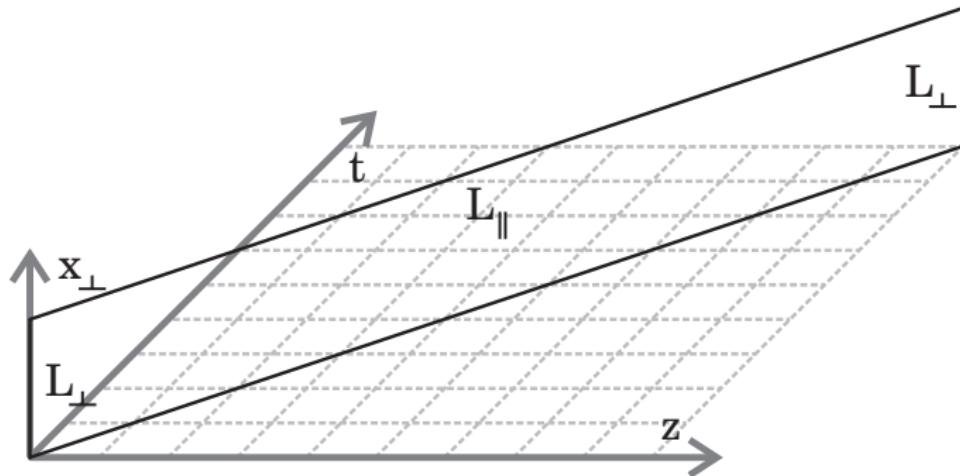
Accounting for the temperature dependence of the gluon condensate  
(N.O. Agasian, '03)  $\Rightarrow$

$$\langle B_i^a(x) B_k^b(x') \rangle = \delta^{ab} \delta_{ij} \frac{\langle (F_{\mu\nu}^a)^2 \rangle \coth(\frac{\mu}{2T})}{12(N_c^2 - 1)} D(x - x').$$

# The jet quenching parameter

$\hat{q}$  is defined through the adjoint Wilson loop as

$$\langle W_{L_{\parallel} \times L_{\perp}}^{\text{adj.}} \rangle = \exp \left( -\frac{\hat{q}}{4\sqrt{2}} L_{\parallel} L_{\perp}^2 \right).$$



The hierarchy of scales:  $\mu^{-1} \ll L_{\perp} \ll \beta \ll L_{\parallel}$ .

# The jet quenching parameter

At  $T > 0$ , the loop splits into strips of the temporal extension  $\beta \Rightarrow$

- a contribution of individual strips

$$\hat{q} = -\frac{4\sqrt{2}n}{L_{\parallel} L_{\perp}^2} \ln \left\langle W_{\text{one-strip}} \right\rangle,$$

where  $n = \frac{L_{\parallel}}{\sqrt{2}\beta}$  is the number of strips:

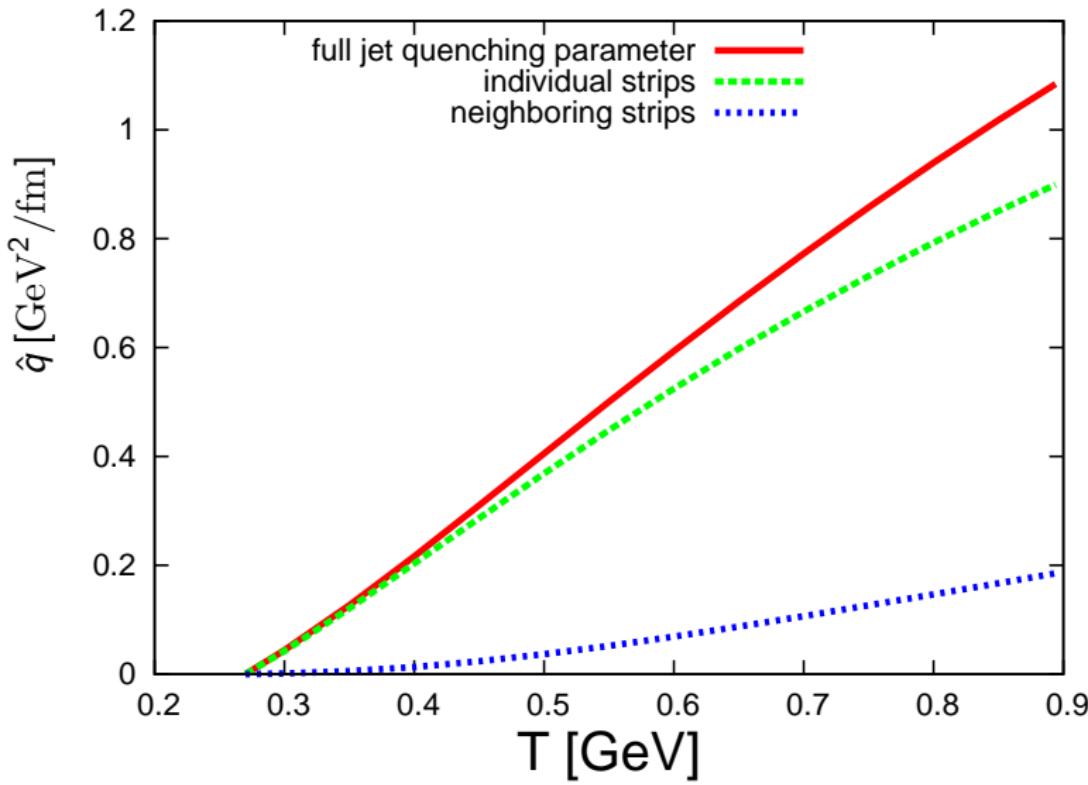
$$\hat{q} = \frac{g^2 \langle (F_{\mu\nu}^a)^2 \rangle}{16\mu} \left[ \sqrt{2} - \frac{1}{\beta\mu} \left( 1 - e^{-\sqrt{2}\beta\mu} \right) \right] \left[ \coth \left( \frac{\mu}{2T} \right) - \coth \left( \frac{\mu}{2T_c} \right) \right],$$

- a contribution of neighboring strips:

$$\Delta \hat{q} = \frac{g^2 \langle (F_{\mu\nu}^a)^2 \rangle e^{-\beta\mu}}{16\mu} \left[ 1 - \frac{1}{\beta\mu} \left( 1 - e^{-\beta\mu} \right) \right] \left[ \coth \left( \frac{\mu}{2T} \right) - \coth \left( \frac{\mu}{2T_c} \right) \right]$$

is suppressed by the factor  $e^{-\beta\mu}$ .

# The jet quenching parameter



# Summary

- Within the stochastic vacuum model, the jet quenching parameter  $\hat{q}$  was calculated in SU(3) quenched QCD at  $T < \mu = 894$  MeV. At these temperatures,  $\hat{q}$  is predominantly defined by the gluon condensate and the vacuum correlation length  $\mu^{-1}$ .
- Numerically, when the temperature varies from  $T_c$  to  $\mu$ ,  $\hat{q}$  rises from zero to 1.1 GeV<sup>2</sup>/fm.
- In the large- $N_c$  limit,  $\hat{q} \sim N_c^0$ , similarly to perturbative QCD and  $\mathcal{N} = 4$  SYM.