# Jet Evolution in the Quark Gluon Plasma

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# Outline of the talk:

- Jet evolution in vacuum in a leading log picture
- Modification of the evolution equations in the quark gluon plasma
- Evolution of the multiplicity, centroid and width of the $dN/\mathrm{d}\log x$ distribution of partons in the jet
- Speculations on further work

# Jets become important at LHC

$10^{11}$/month

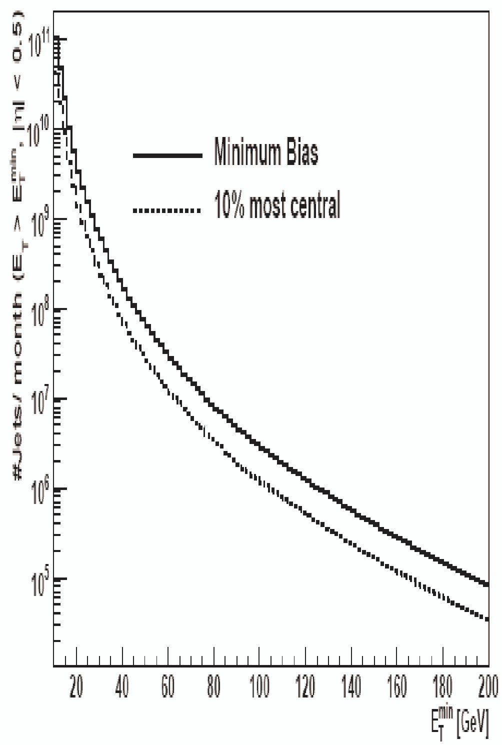



**Figure 6.2:** Number of jets with $E_T > E_T^{min}$ and $|\eta| < 0.5$ produced in Pb–Pb collisions in one effective month of running ($10^6$ s). The minimum bias rate (solid line) is compared to the rate in 10% most central collisions (dashed line).

100/event

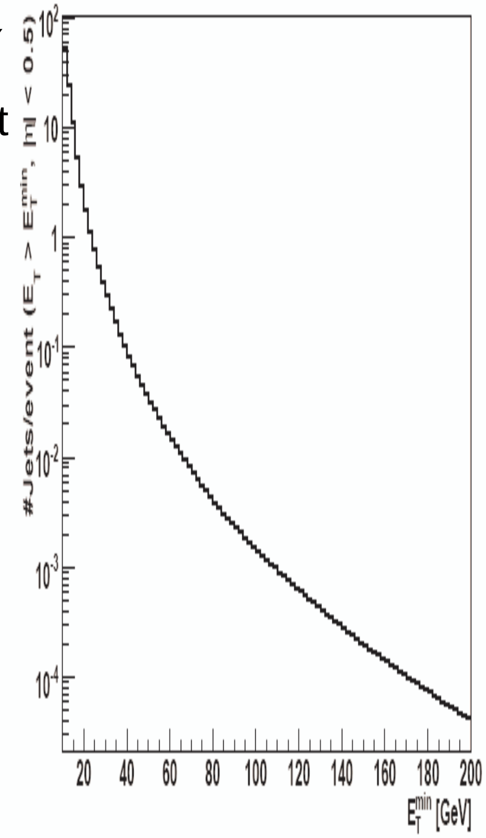



**Figure 6.3:** Average number of jets with $E_T > E_T^{min}$ and $|\eta| < 0.5$ per event in the 10% most central Pb–Pb collisions.

# Parton evolution by branching processes

- We use a distribution function which has three variables:
- Momentum fraction *z*, the virtuality (mass squared) $Q^2$ and the transverse momentum $pt^2$.

$$D_i^j(z, Q_0^2, \vec{p}_t) = \delta(i,j)\delta(x-1)\delta^2(p_T) \tag{18}$$

and the equation is:

$$Q^2 \frac{D_i^j(z, Q^2, \vec{p}_t)}{\partial Q^2} = \tag{19}$$

$$= \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u} P_i^r(u, \alpha_s(Q^2)) \frac{d^2\vec{q}_t}{\pi} \delta(u(1-u)Q^2 - Q_0^2/4 - q_t^2) D_r^j(z/u, Q^2, \vec{p}_t - z/u\vec{q}_t)$$

# Solution of evolution in vacuum

Next we define the Mellin Transform of the equation:

$$d(J,Q^2) = \int_0^1 dz D(z,Q^2) z^{J-1}$$

$$d(J,Q^2) = c(\frac{Q^2}{Q_0^2})^{\gamma(\alpha,J)} =$$

$$c\exp[\gamma log(Q^2/Q_0^2)] = = c\exp[\int_{Q_0^2}^{Q^2} dQ'^2 \gamma(J,\alpha)/Q'^2]$$

- First, we use the $p_t$ integrated distribution, and go over to the Mellin Transform
- Then, we make an ansatz with an anomalous dimension $\gamma$ ($\alpha_{s,}$)
- $\gamma$ ($\alpha_s$) obeys a quadratic equation

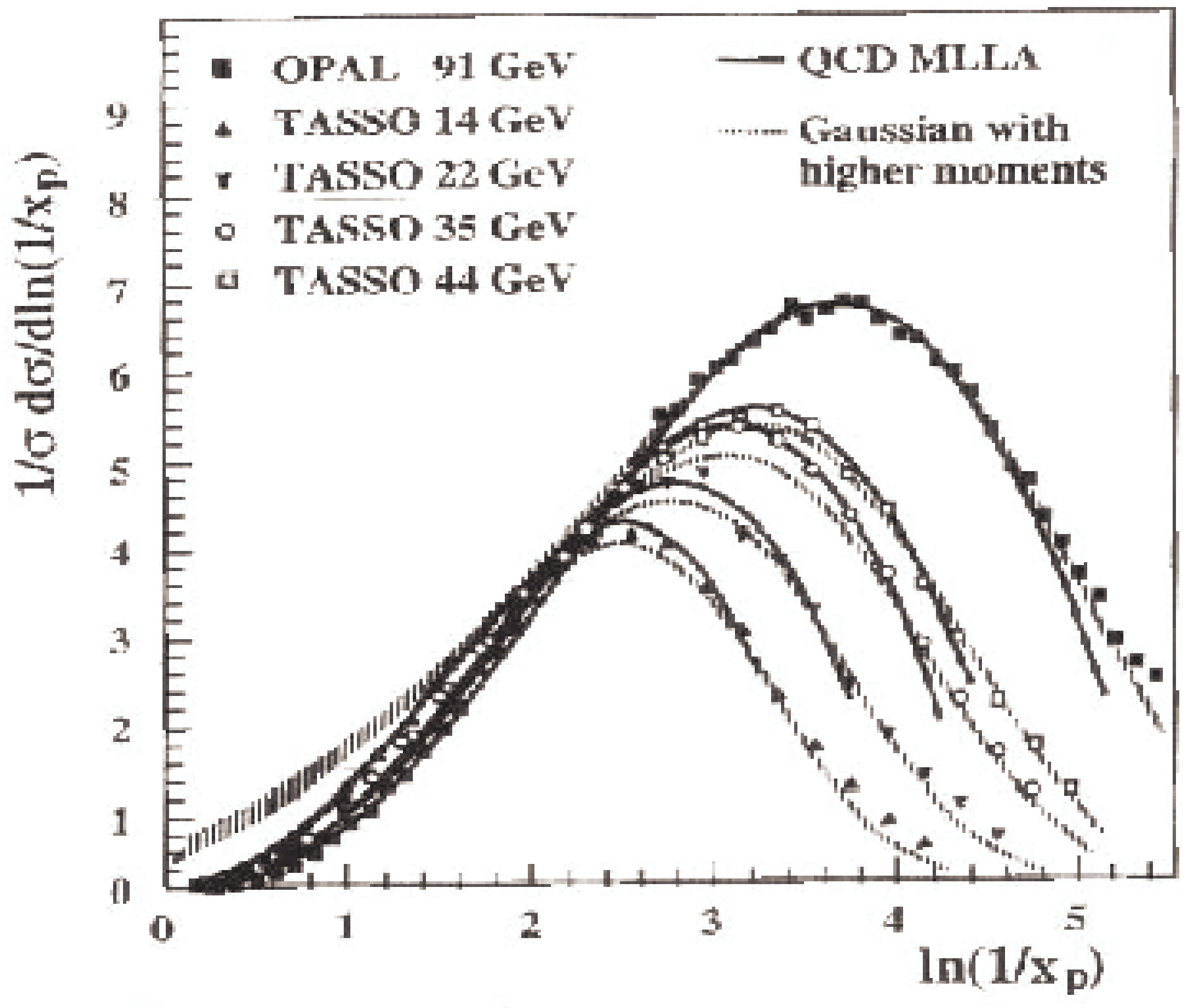
# Concentrate on small *z*-behavior

$$\gamma(J,\alpha) = $$
$$= \frac{\alpha_s(Q^2)}{2\pi}\int_0^1 du 2C_A u^{J-2+2\gamma}$$

$$n(Q^2) = c exp[1/b\sqrt{\frac{2C_A}{\alpha_s \pi}}]$$
$$b = \frac{11}{4\pi}$$

- Evolution is limited to the dominant gluon-gluon splitting *P(u) =c/u*
- Suppression of soft gluon emission due to coherence, which is reflected in the exponent
- Resulting multiplicity n(Q) in vacuum after integration over $Q^2$ or $\alpha_s(Q^2)$
- Multiplicity increases with virtuality

# Differential multiplicity distribution *dN/d* ln*(1/x)* in a jet of (90 GeV)²



# What happens in the quark gluon plasma ?

- The parton can make collisions with the plasma particles
- The density of plasma particles $n_g=2\ T^3$
- The cross section is determined by Debye-screened gluon exchange
- The lifetime for a collision is estimated $t=E/Q_0^2$, where we use as an upper bound the $Q^2$ of the dominant lowest virtuality gluons. $Q_0$=2 GeV is the infrared cutoff

# Modification of evolution equation in the quark gluon plasma

$$Q^2 \frac{\partial D_i^j(z,Q^2,\vec{p}_t)}{\partial Q^2} =$$

$$= \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 \frac{du}{u} P_i^r(u,\alpha_s(Q^2)) \frac{d^2\vec{q}_t}{\pi} \delta(u(1-u)Q^2 - Q_0^2 - q_t^2) D_r^j(z/u, Q^2, \vec{p}_t - z/u\vec{q}_t) + S(Q^2,\vec{p}_t) \tag{23}$$

with the scattering term $S(Q^2, p_t)$, note The Lorentz factor is really $zE$ .

$$S(Q^2,\vec{p}_t) = zE/Q_0^2 n_g \int_z^1 dw \int d^2\vec{q}_t \frac{d\sigma_i^r}{d^2\vec{q}_t} (D_r^j(w,Q^2,\vec{p}_t - w\vec{q}_t) - D_r^j(z,Q^2,\vec{p}_t)) \delta(w - z - \frac{q_t^2}{2m_g E}). \tag{24}$$

As before, we limit ourselves to the gluon cascade

# Method of solution is similar to the vacuum case

- The scattering term shows up as a drift term to smaller *z* value

$$\hat{S}(Q^2) = \frac{zEn_g\sigma_i^r < p_t^2 >}{Q_0^2 2m_g E}\frac{\partial D_r^j(z, Q^2)}{\partial z}$$

We read off a small parameter $\delta$, which is scale invariant , i.e. no longer dependent on the energy and defined as:

$$\delta = \frac{zn_g\sigma_i^r < p_t^2 >}{2m_g Q_0^2} \tag{29}$$

$$\sigma_g^g(Q^2, T) = \frac{9\pi\alpha(Q^2)^2}{2m_D^2} \tag{30}$$

The mean transverse momentum squared I assume is near the Debye mass squared.

$$< p_t^2 > \approx m_D^2 \tag{31}$$

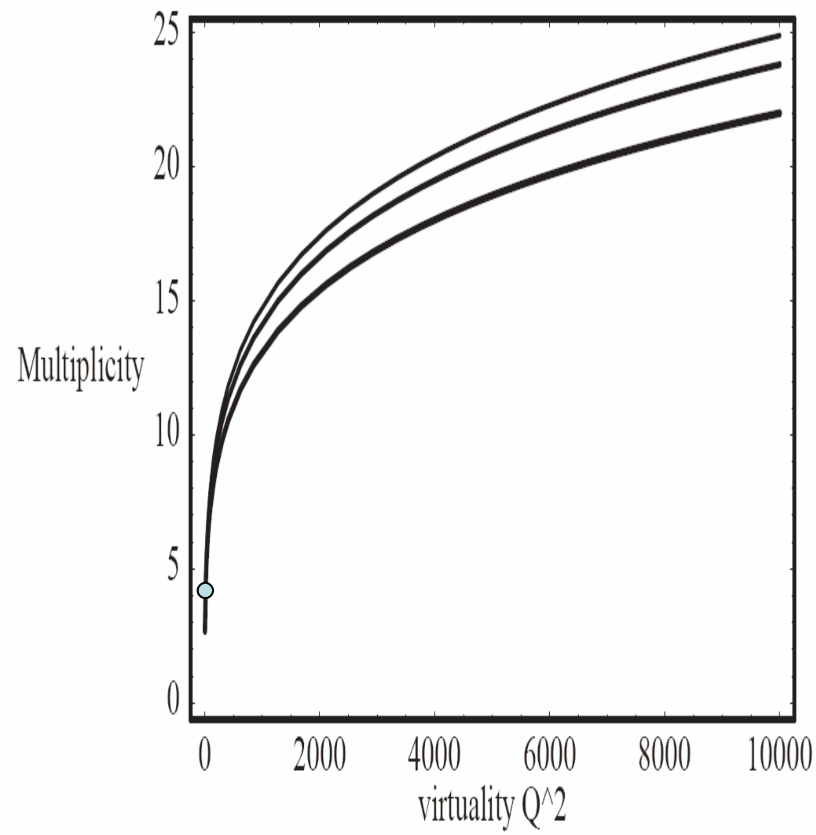
# Subleading correction to the DGLAP evolution

- The scattering term *S* leads to a subleading correction to the dominant $\alpha_s$ evolution. The medium modification term is proportional to $\alpha_s^{(3/2)}$
- Its temperature dependence is $T^2$

$$z\frac{T^2}{Q_0^2}\frac{3}{2}\sqrt{\pi}\alpha(Q^2)^{3/2}$$

# Mean Multiplicity in the Quark Gluon Plasma at temperature *T*

- Mean multiplicity is growing with increasing temperature for *T*=0.8 GeV and *T*=1.0 GeV
- The total multiplicity in vacuum is fixed with $e^+e^-$ data
- The medium multiplicity at the starting scale of evolution, i.e. in the infrared, is fixed to be equal to the vacuum value



# Centroid of the Gaussian Distribution in ln(1/*x*)

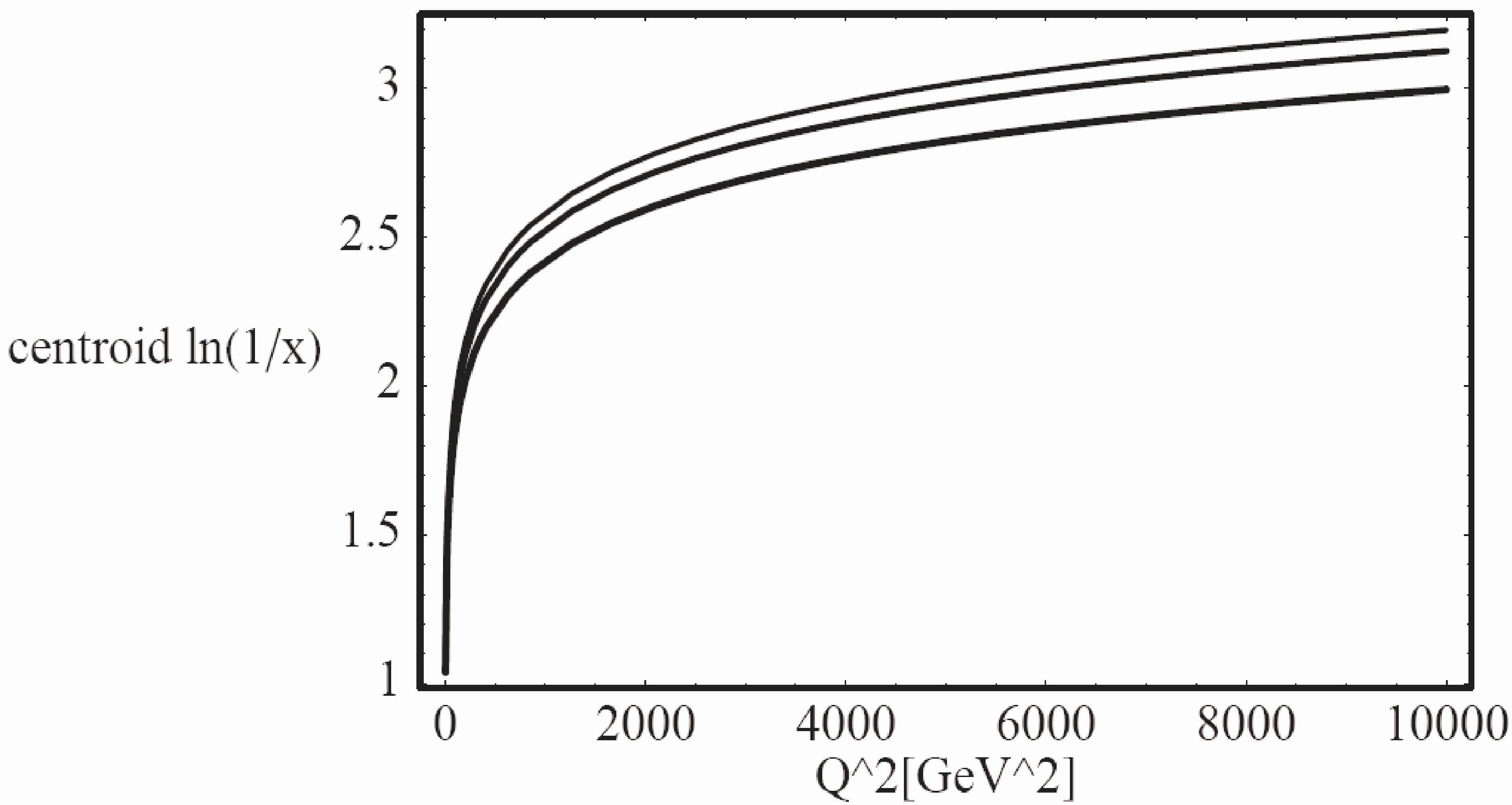



Figure 2: Centroid of the $ln(1/x)$ distribution in vacuum (lowest curve),at $T = 0.8GeV$ (middle curve) and at $T = 1.0GeV$ (top curve)

# Width of the in-medium ln(1/*x*) distribution

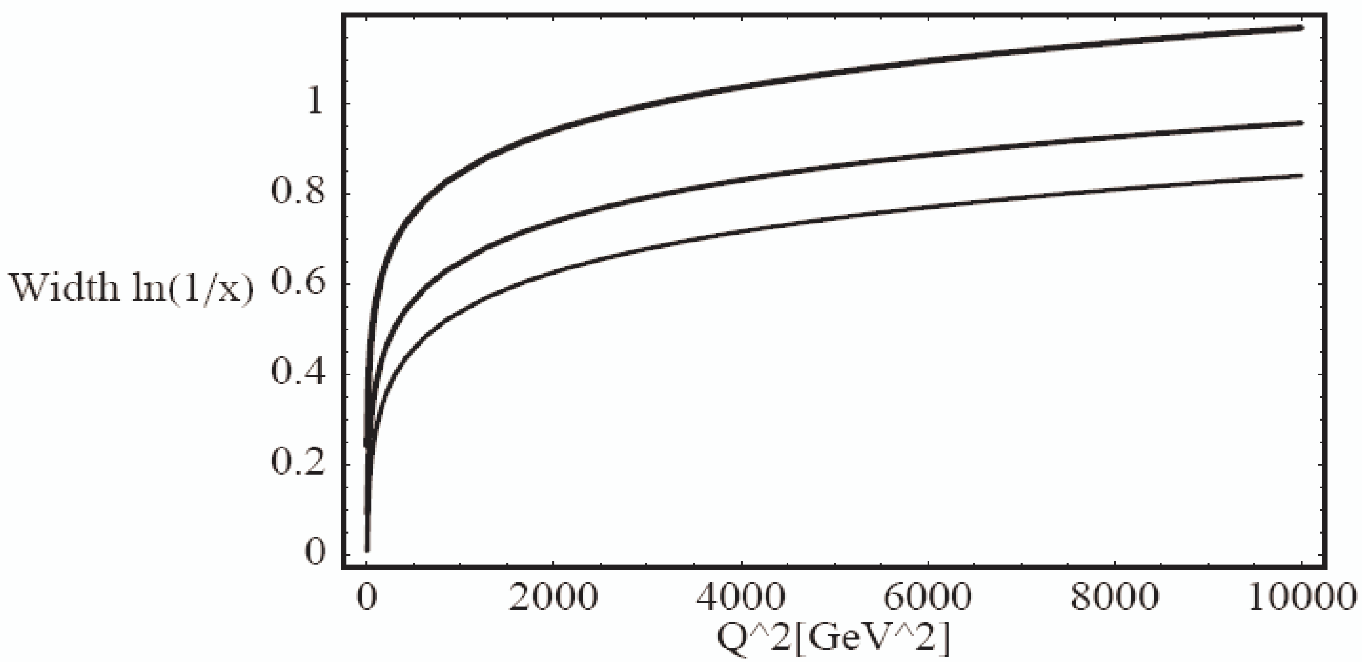



Figure 3: Width of two jet $ln(1/x)$ distribution with invariant mass $Q^2$ in vacuum (top ! curve), at $T = 0.8GeV$ (middle curve) and at $T = 1.0GeV$ (lowest curve)

# Jet particle spectrum in vacuum and in the quark gluon plasma

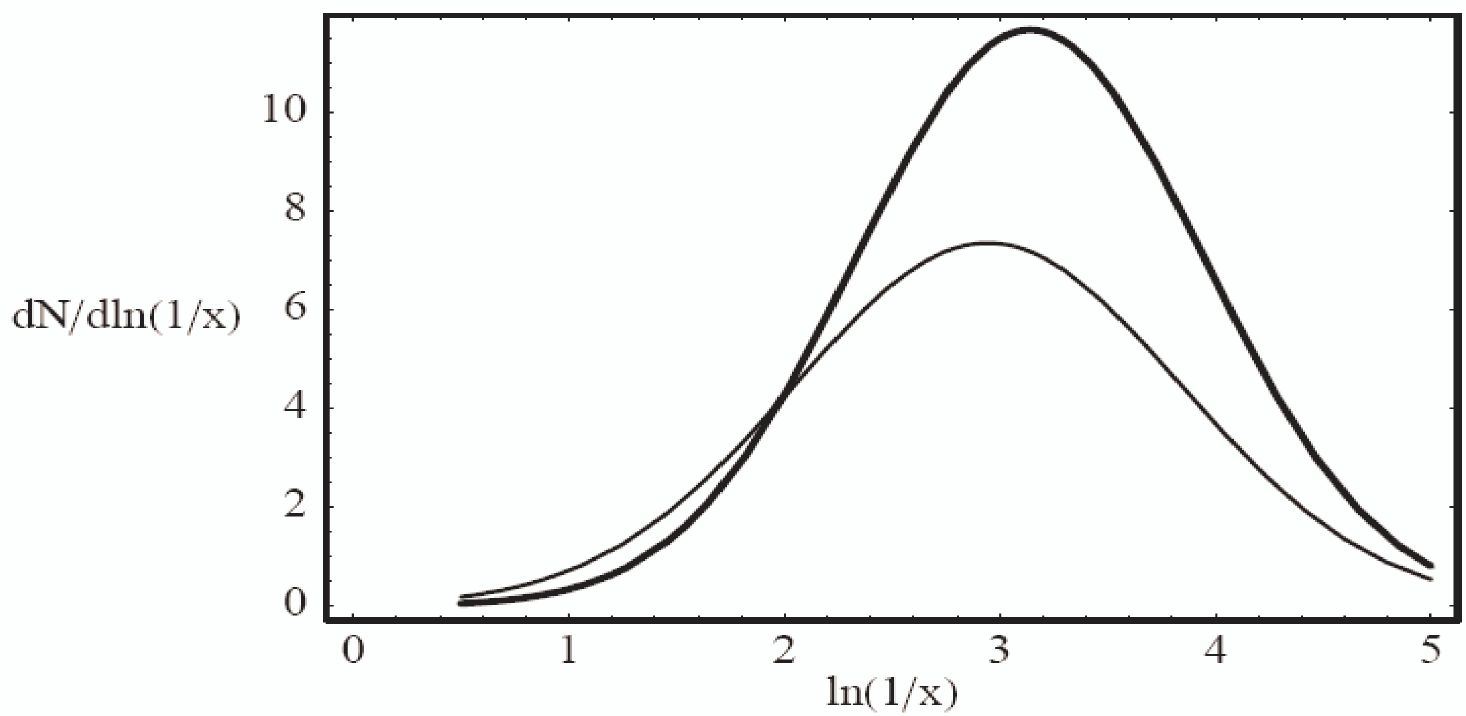



Figure 5: Differential multiplicity $dN/dln(1/x)$ of jet particles inside a jet with invariant mass $Q^2 = 90GeV^2$ in vacuum (fine drawn curve), at $T = 1.0GeV$ (full curve)

# Conclusions

- The leading log approximation gives a qualitative picture of jet evolution in the plasma
- Results of evolution equations show a higher multiplicity shifted to lower *x* with a narrower distribution
- They are qualitatively similar to the results of Borghini and Wiedemann (Nucl. Phys. A 774 (2006) 540), but they can be calculated parametric as a function of plasma density or temperature
- With this calculation the jet measurement becomes a tool of diagnosis for the quark gluon plasma
- Necessary to calculate the Monte Carlo cascade in fragmentation together with collisions (K. Zapp, S. Domdey)
- Challenging new questions regarding the input cross sections and medium effects.

# Method of full solution:

$$
\begin{aligned}
d(J,Q^2) &= c(\frac{Q^2}{Q_0^2})^{\gamma(\alpha,J)} = \\
c\exp[\gamma log(Q^2/Q_0^2)] = &= c\exp[\int_{Q_0^2}^{Q^2} dQ'^2\gamma(J,\alpha)/Q'^2]
\end{aligned}
$$

- Make a Taylor expansion of the anomalous moment around *J=1*, the dominant Mellin amplitude.
- The inverse Mellin transform from the integration in the complex *J* plane along the imaginary axis gives a Gaussian multiplicity distribution in ln*(1/x)*
- The zeroth order term is the normalization, the linear term gives the centroid and the quadratic the width of the Gaussian distribution in ln *(1/x)*

# High $p_T$-suppression, due to parton energy loss ?

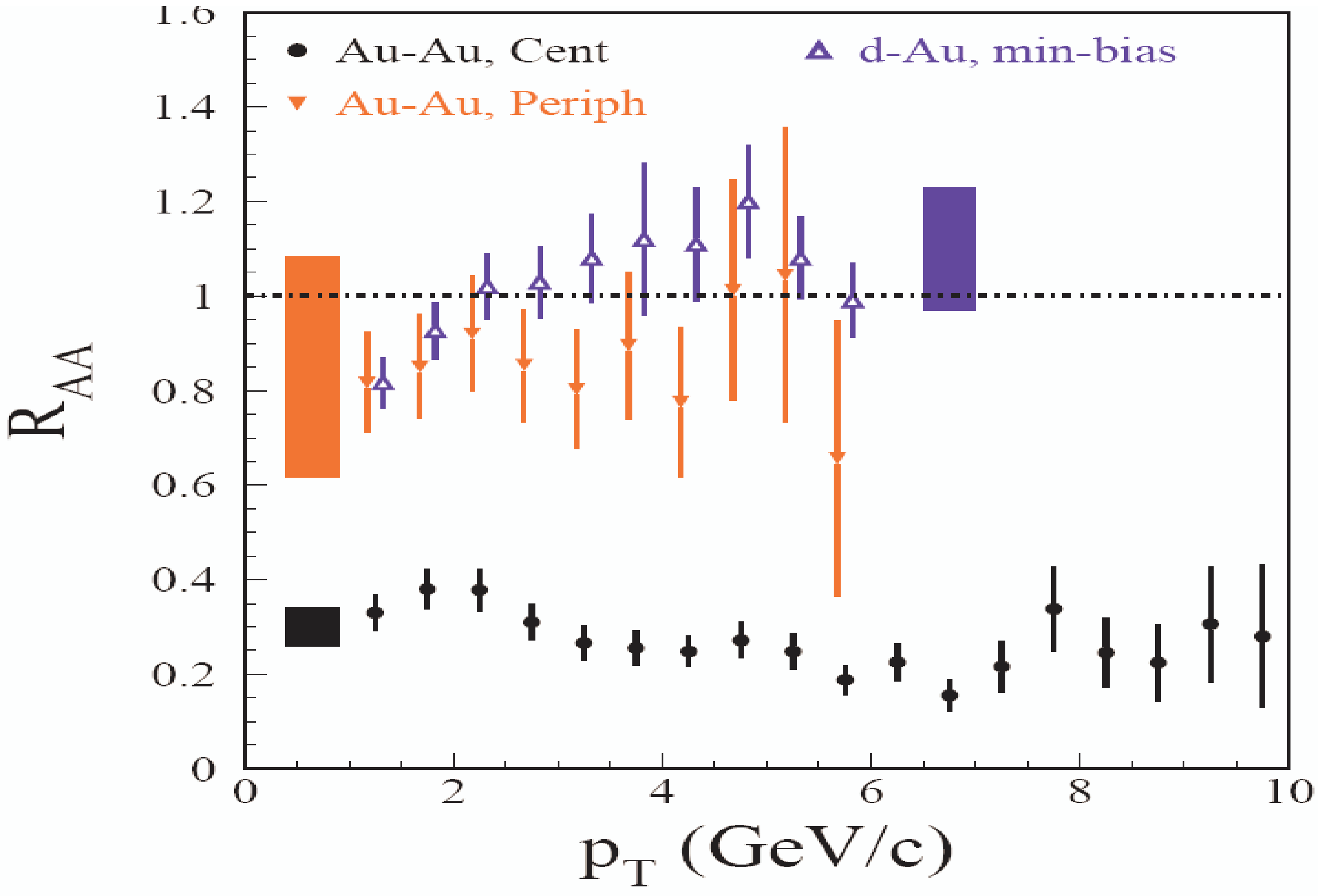
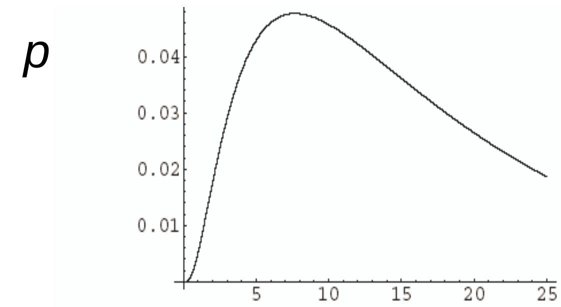



Fig. 36. $\pi^0$ $R_{AA}(p_T)$ for central (0–10 %) and peripheral (80–92 %) Au+Au collisions

# Space time development (Initial virtuality $t_0$=100 Gev$^2$ → $t_1$)




t1

Out[45]= - Graphics -

▪ Probability Distribution of radiated virtualities t1 when original virtuality is t0=100 GeV^2

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In[46]:= Plot[{.2*T[t]}, {t, 0.1, 22}]
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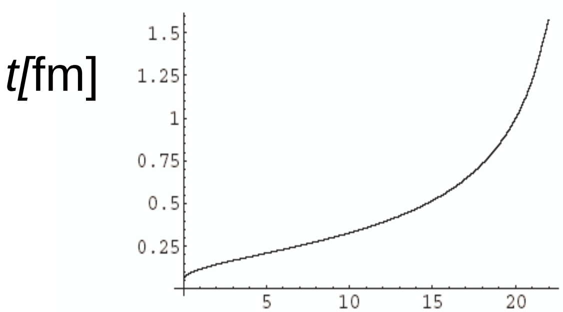


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▪ Mean Time in fm for radiation
as a function of radiated virtuality t1 [GeV^2]

Take RHIC case:
Mean final virtuality of radiated gluons is $t_1$=10 GeV$^2$

Mean time for radiation
$\langle t \rangle$=0.7 fm/$c$

# Space time Structure of hadron production

- Induced radiation and fragmentation can not be separated
- The produced parton has time like virtuality and loses energy even in vacuum ( vacuum energy loss).
- No difference in decay time between charm quarks and light quarks, because of high initial virtuality
- Most descriptions treat first the energy loss of an on shell quark in the medium and then hadronization,as below

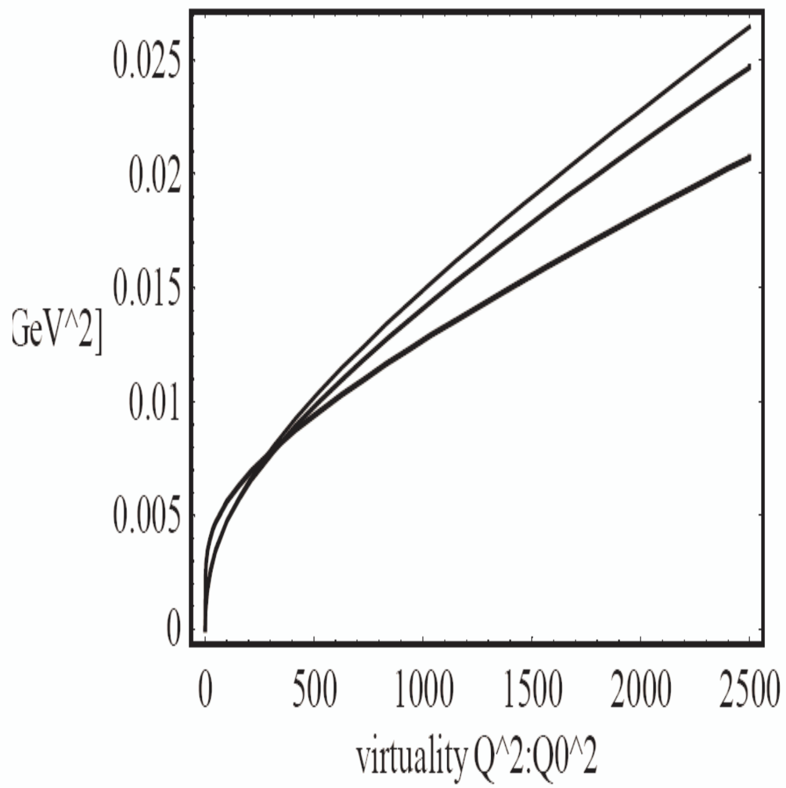
$$z_c D'_{h/c}(z_c, Q_c^2) = z'_c D_{h/c}(z'_c, Q_{c'}^2) + N_g z_g D_{h/g}(z_g, Q_g^2) \ ;$$

$$z'_c = \frac{p_h}{p_c - \Delta E_c(p_c, \phi)} \ , \quad z_g = \frac{p_h}{\Delta E_c(p_c, \phi)/N_g} \ ,$$

Modification of fragmentation function separated from energy loss is not justified

# Mean $p_t^2$ of the jets

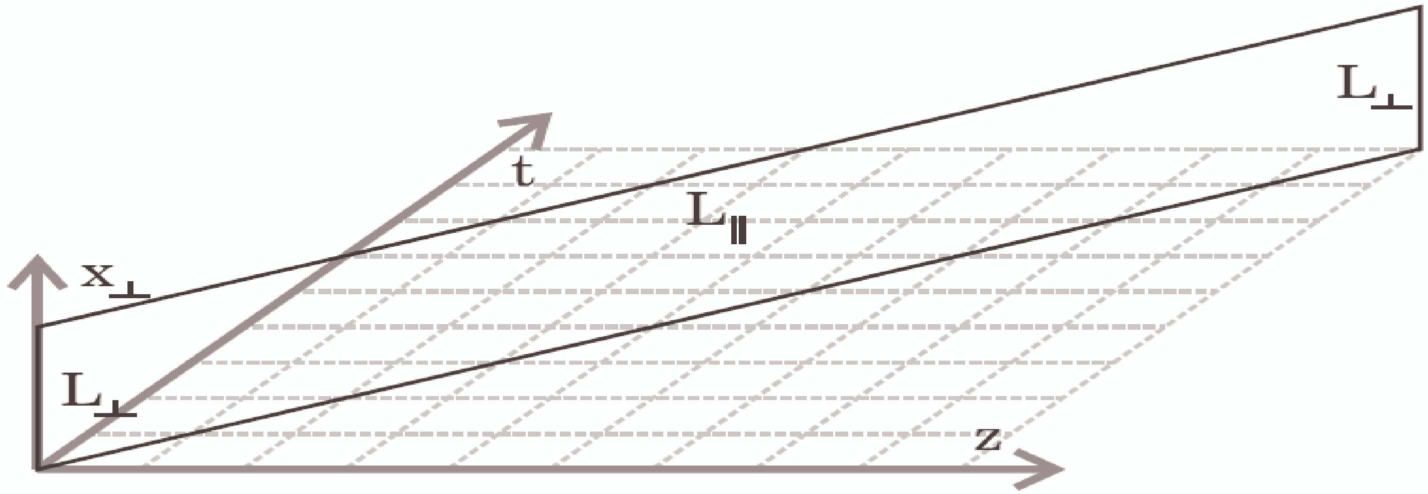
- With the help of the triple differential distribution very detailed questions can be analyzed
- Increase from $p_t^2 = 0$ at infrared scale $Q_0^2$=4 GeV$^2$
- Top curve is for T=0, middle T=0.2 GeV and bottom for T=0.4 GeV
- An improved method will use Monte Carlo Methods to get solutions, instead of using the Gaussian parametrisation



# Jet quenching q in a Wilson Line Calculation comes out too small

- Calculation of Antonov, Dietrich and HJP
- q<1 GeV$^2$ /fm for T<1 GeV

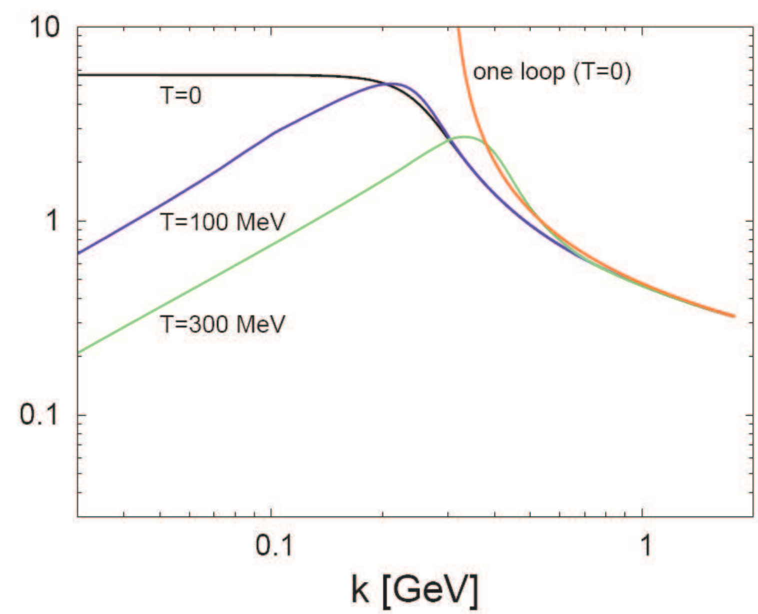
$$\left\langle W^{\mathrm{adj.}}_{L_{\parallel}\times L_{\perp}} \right\rangle = \exp\left( -\frac{\hat{q}}{4\sqrt{2}} L_{\parallel} L_{\perp}^{2} \right) .$$



# Medium induced scattering

- Mean free path is shorter due to larger coupling α(*k*,*T*)
- Debye mass can be determined self-consistently from strong coupling α(*k*,*T*)
- Running α(*k*,*T*) at finite temperature is calculated from RG equation (J. Braun, H. Gies, **hep-ph/0512085)**

$$d\sigma_i/dq_{\perp i}^2 \approx C_i \frac{4\pi\alpha^2}{(q_{\perp i}^2+\mu^2)^2}$$



# Plan: Try to calculate Wilson loop with field strength correlators

$\backslash \cos\theta /$

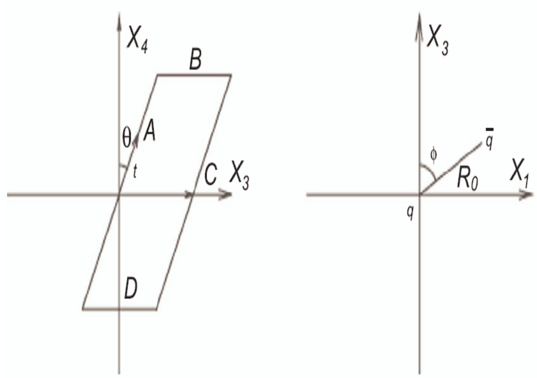



Figure 1: Configuration of the Wegner-Wilson loop in Euclidean space-time.

$$
\begin{aligned}
< W[C] > &= e^{-\sigma R_0 \alpha T} \\
\sigma &= \frac{\pi^3 G_2 a^2 \kappa}{18},
\end{aligned}
$$

$$\alpha^2 = 1 - \cos^2\phi \sin^2\theta .$$

$$
\begin{aligned}
Area &= T R_0 \int_{-1/2}^{1/2} du \int_0^1 dv \sqrt{\left(\frac{dX_\mu}{du}\right)^2 \left(\frac{dX_\mu}{dv}\right)^2 - \left(\frac{dX_\mu}{du}\frac{dX_\mu}{dv}\right)^2} \\
&= T R_0 \alpha .
\end{aligned}
$$

# Inverse Mellin Transform

- Integration over J in the complex plane can be done analytically
- Gaussian integral

The inverse Mellin transform is calculated as follows with $\tilde{J} = -i(J-1)$:

$$D(z,Q^2) = \frac{C}{2\pi i}\int_{1-i\infty}^{1+i\infty} dJ \frac{1}{x^J} Exp[a(J-1)^2 - b(J-1)] = \frac{C}{2\pi}\int_{-\infty}^{+\infty} d\tilde{J} Exp[-a\tilde{J}^2 + i\tilde{J}(-b + Ln[1/x])] \tag{41}$$

The lowest order coefficient of the $J-1$ expansion is related to the multiplicity, the next coefficient to the center of gravity of the distribution in $Log(1/x)$ of the generated gluons, and the quadratic coefficient to the width of the distribution in $ln(1/x)$.

# Energy loss can be related to the expectation value of a Wilson loop in thermal configuration

$$\frac{1}{N^2-1}\langle\langle \mathrm{Tr}\left[W^{A\,\dagger}(\mathbf{y})\,W^A(\mathbf{x})\right]\rangle\rangle_t \approx \exp\left[-\frac{C_A}{4\,C_F}\int d\xi\, n(\xi)\,\sigma(\mathbf{x}-\mathbf{y})\right]$$

$$\approx \exp\left[-\frac{(\mathbf{x}-\mathbf{y})^2}{8}\frac{C_A}{C_F}Q_s^2\right]\,.$$

The two Wilson lines are given by the quark in the amplitude *T* and the quark in the complex conjugate amplitude *T**. This artificial pair forms a dipole which can be handled with standard methods. For a homogeneous medium the integral over the traversed length gives the length $L^-$, and $L^2$ is the dipole size.

$$\langle W^A(\mathcal{C})\rangle \;\approx\; \exp\left[-\frac{1}{4\sqrt{2}}\hat{q}\,L^-\,L^2\right]$$