Perturbative jet energy loss mechanisms

Learning from RHIC, extrapolating to LHC

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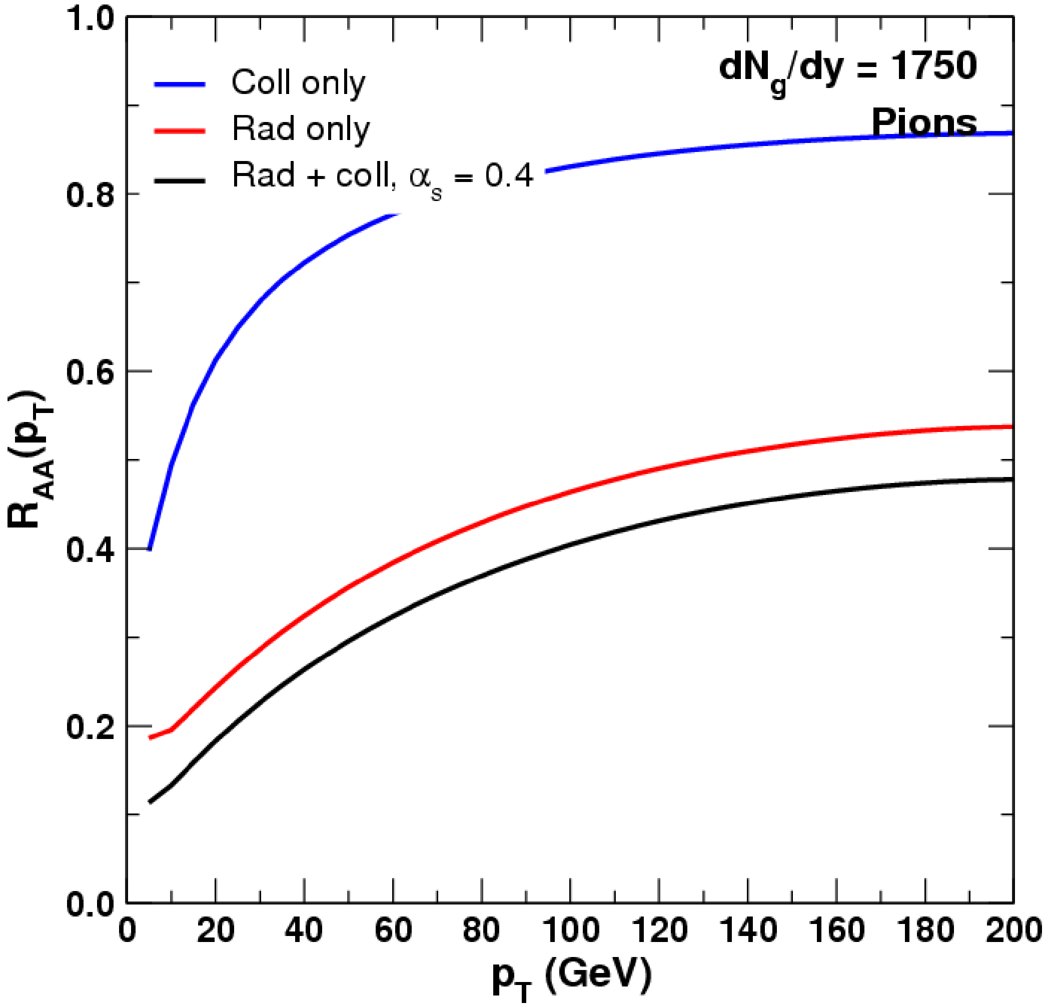
Collisional unimportant at LHC?

Note: dNg/dy = 2900 will come later



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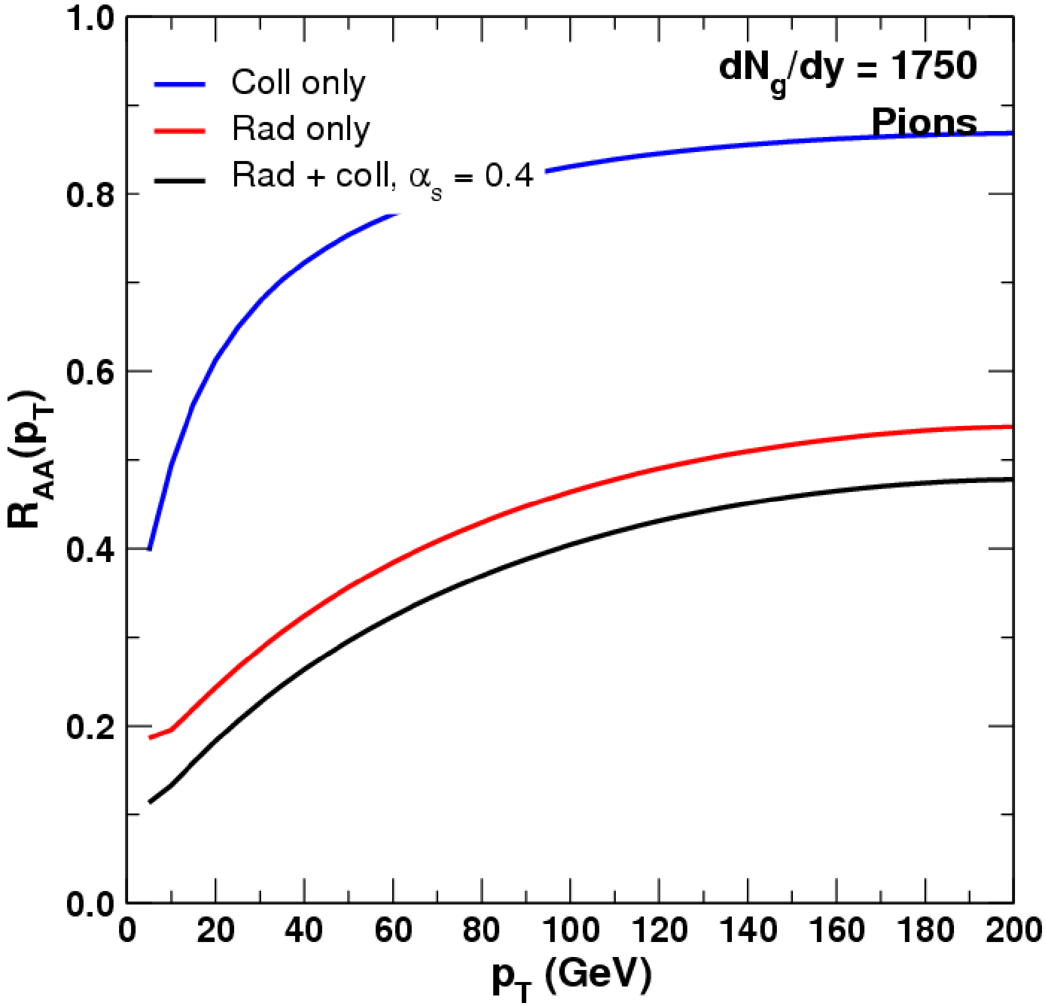
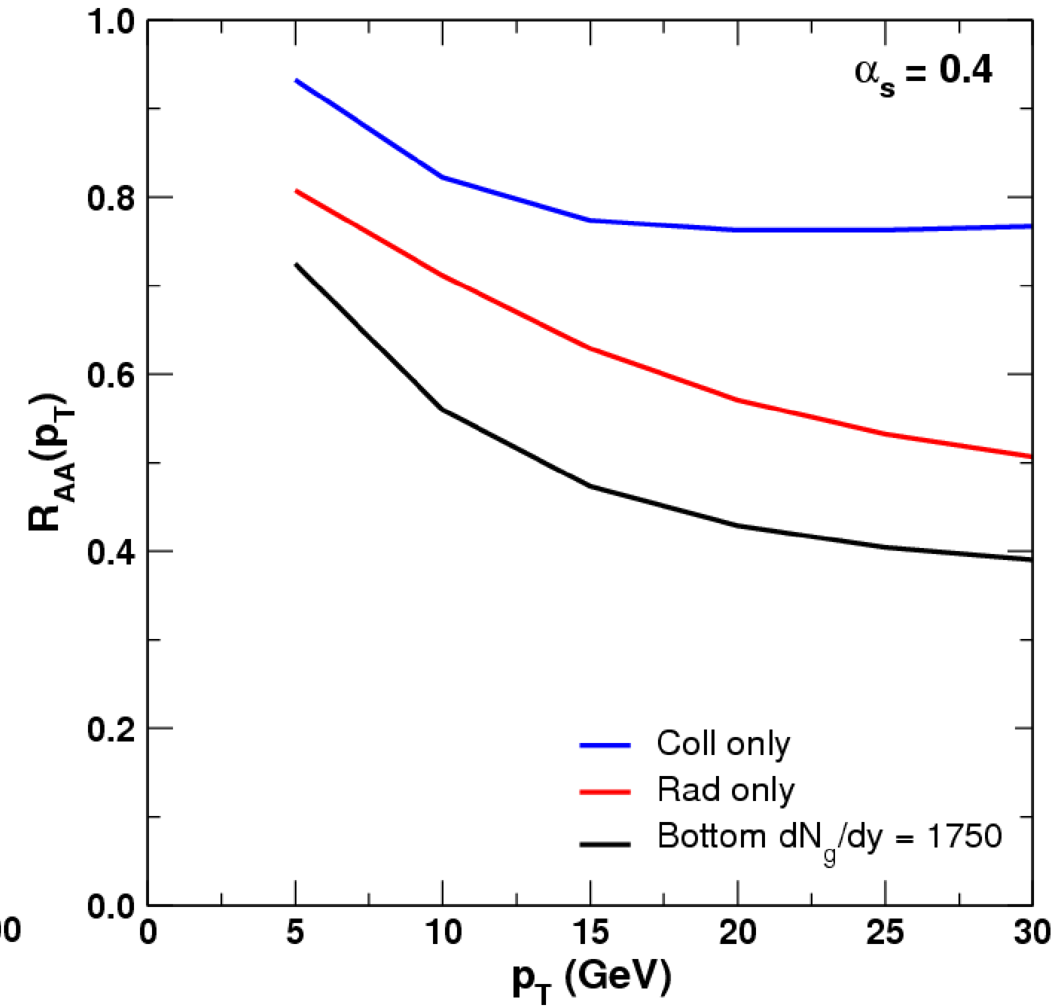




Why look at collisional processes?

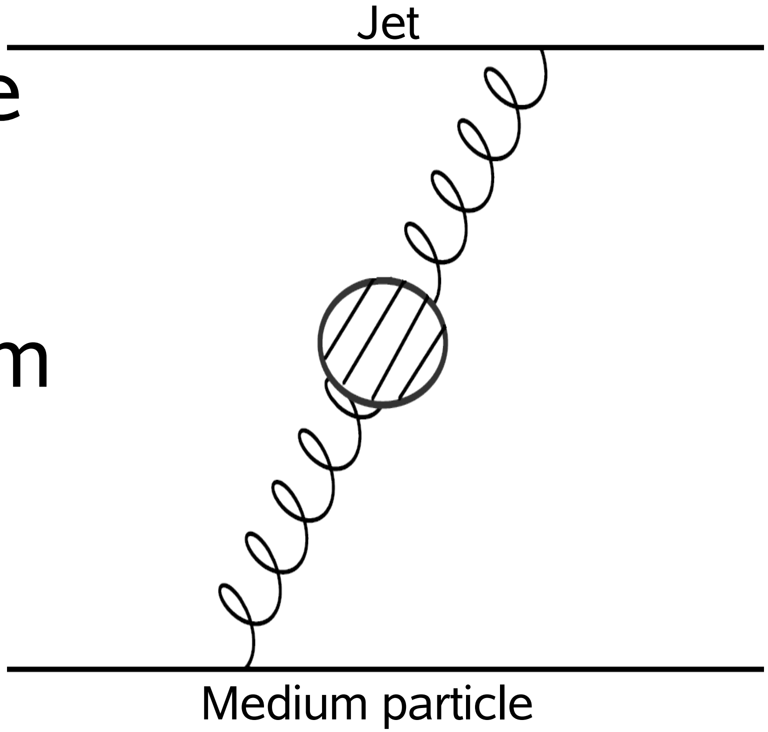
1) Know what are the energy loss mechanisms are.
Different energy loss mechanisms scale differently with density and jet energy.
Consistency between RHIC and LHC?

2) Collisions are what induce (cause the medium modification to) the radiative energy loss.

The model

Hybrid radiative + collisional model

- Incoherent addition of (D)GLV radiative with collisional energy loss model
- Collisional: t-channel exchange
- HTL modified gluon propgator
- Scattering off massless medium
- Neglect difference between Q-q and Q-g scattering (except Casimir)
- Neglect finite time effects



Collisional energy loss formalism

$$\frac{dN}{d\omega} = \frac{1}{E^2}\frac{1}{v}\frac{1}{(2\pi)^4}\int_{|p-\sqrt{(\omega+E)^2-M^2}|}^{p+\sqrt{(\omega+E)^2-M^2}} dq \int_{\frac{1}{2}(q+\omega)}^{\infty} dk \int_0^{2\pi} d\phi\,(\ldots)$$

$$\delta(\omega + E - E') = \frac{E'}{pq}\delta\left(\cos\theta_{pq} - \left(\frac{\omega}{vq} + \frac{\omega^2 - q^2}{2pq}\right)\right)$$

$$\delta(\omega - E_k + E'_k) = \frac{E'_k}{kq}\delta\left(\cos\theta_{kq} - \left(\frac{\omega}{v_k q} - \frac{\omega^2 - q^2}{2kq}\right)\right)$$

$$D_{\mu\nu}(q) = Q_{\mu\nu}(q)\Delta_L(q) + P_{\mu\nu}\Delta_T(q)$$

$$Q_{00} = 1 \quad , \quad \Delta_L(Q) = \frac{1}{q^2 - \Pi_l}$$

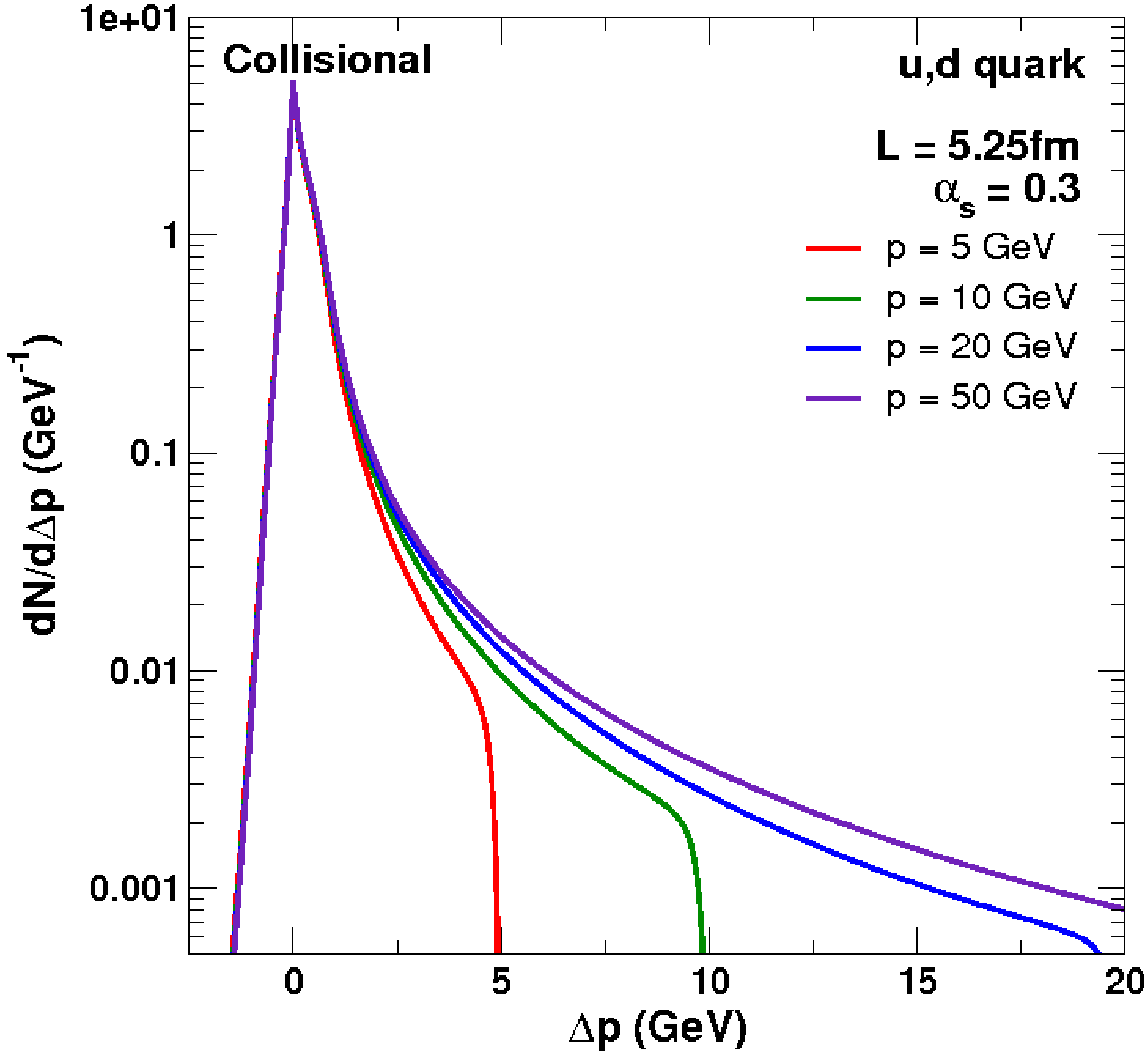
$$P_{ij} = -(g_{ij} + \hat{q}_i\hat{q}_j) \quad , \quad \Delta_T(Q) = \frac{1}{\omega^2 - q^2 - \Pi_t}$$

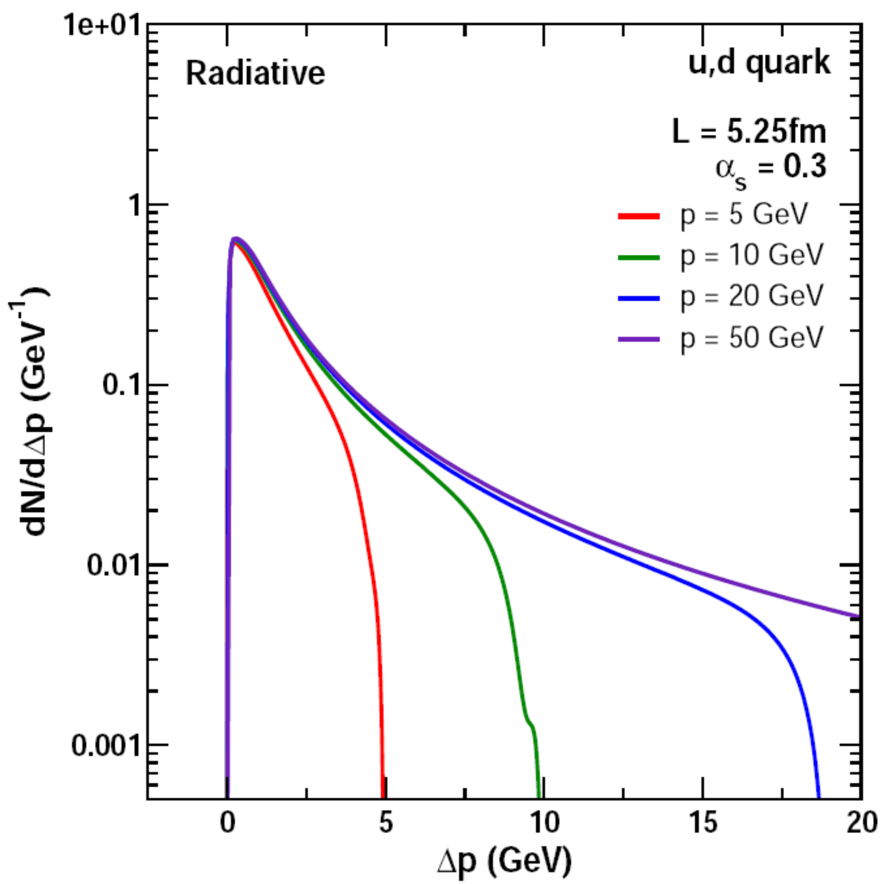
$$\langle |M|^2 \rangle = 16 g^4 k_{CF} E^2 E_k^2 \left[C_{LL}|\Delta_L(q)|^2 + 2C_{LT} Re(\Delta_L(q)\Delta_T^*(q)) + C_{TT}|\Delta_T(q)|^2\right]$$

$$\begin{aligned}
C_{LL} &= \left((1+\frac{\omega}{2E})^2 - \frac{q^2}{4E^2}\right)\left((1-\frac{\omega}{2E_k})^2 - \frac{q^2}{4E_k^2}\right) \\
C_{LT} &= 0 \\
C_{TT} &= \frac{1}{2}\left(v^2 - \frac{\omega^2}{q^2}\left(1+\frac{\omega}{2E}\right)^2 + \frac{1}{E}\left(\omega + \frac{q^2}{4E}\right)\right)\left(v_k^2 - \frac{\omega^2}{q^2}\left(1-\frac{\omega}{2E_k}\right)^2 - \frac{1}{E_k}\left(\omega - \frac{q^2}{4E_k}\right)\right)
\end{aligned}$$

Collisional energy loss distribution

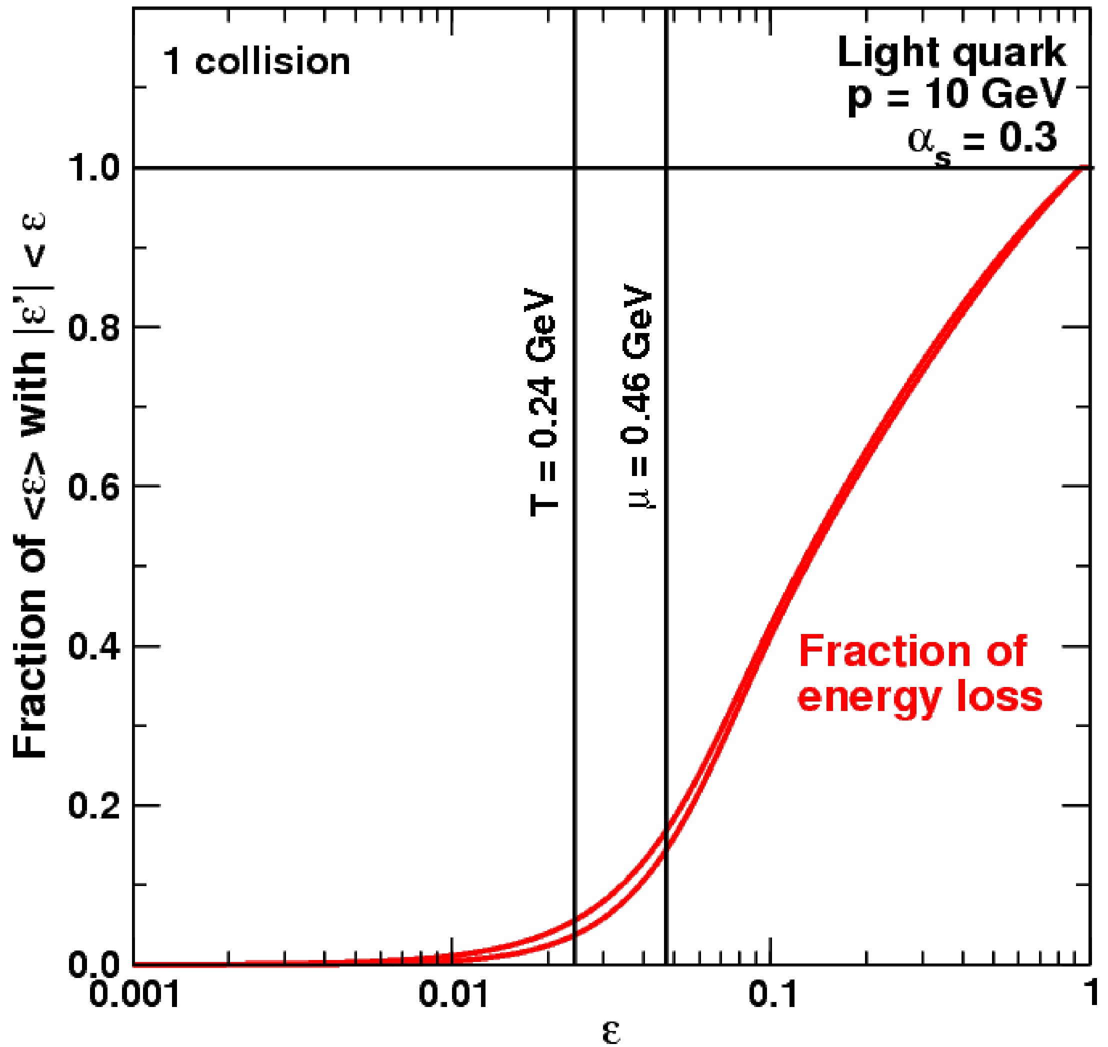
(before multiple collision convolution)



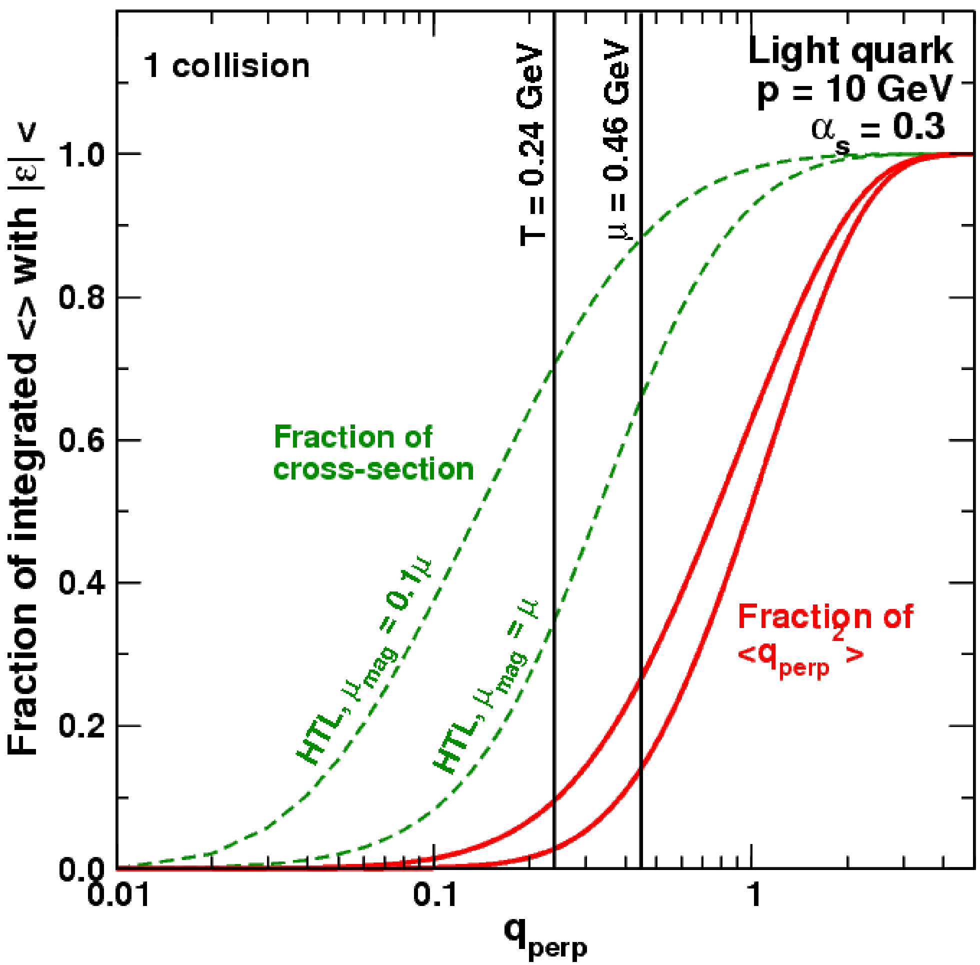


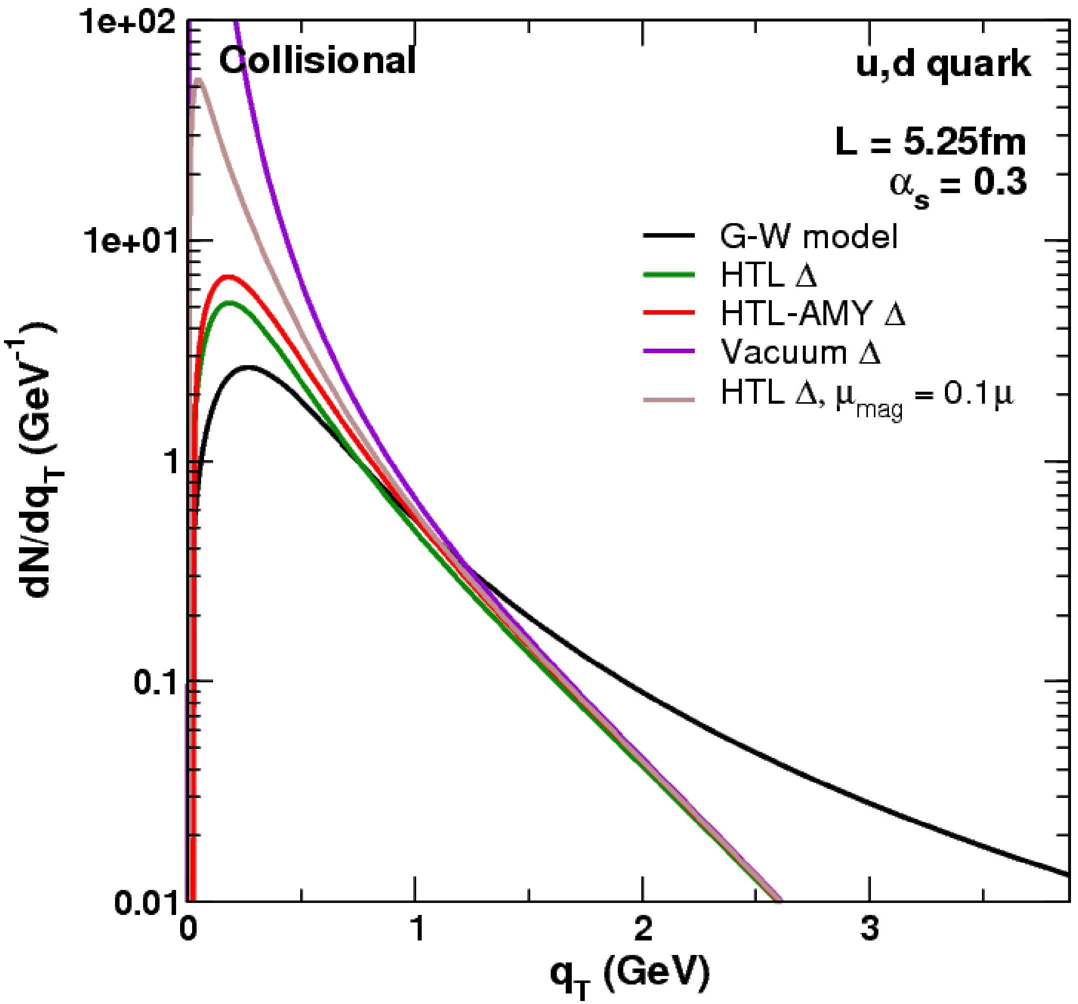
Multiple collisions

- The rare, hard collisions contribute to <ΔE>
- ie NOT well described by continuum Langevin / Fokker-Planck process.



What about q_{perp} distributions?





The rare, hard collisions contribute most to $<q_{perp}^2>$

Transverse diffusion process?

Multiple collisions

- Multiple independent collisions
  Poisson convolution
- Use phenomenological magnetic screening mass

$$P(\epsilon) = \sum \frac{\chi^n e^{-\chi}}{n!} P_n(\epsilon)$$

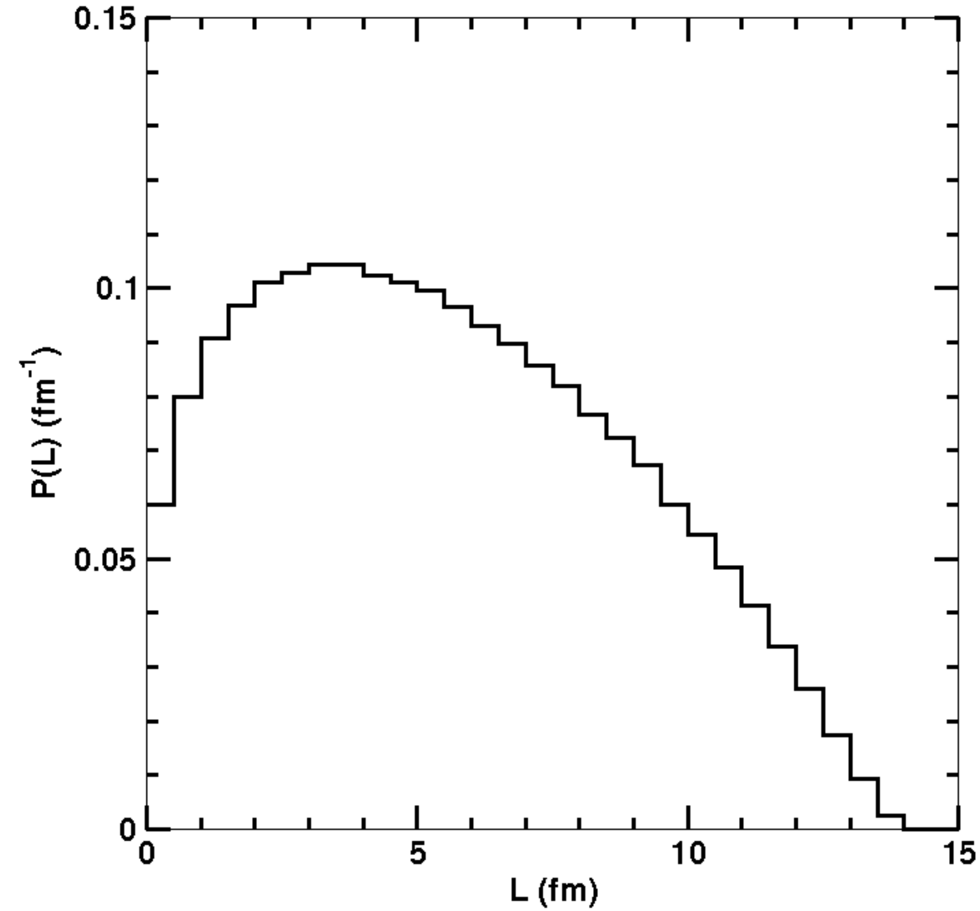
$$\chi = \langle n(L) \rangle = \int_{-\infty}^{E} d\omega \, \frac{dN}{d\omega} \frac{L}{v}$$

$$P_1(\epsilon) = \frac{1}{\chi} \frac{dN}{d\omega}$$

$$P_n(\epsilon) = \int dx \, P_{n-1}(x) P_1(\epsilon - x)$$

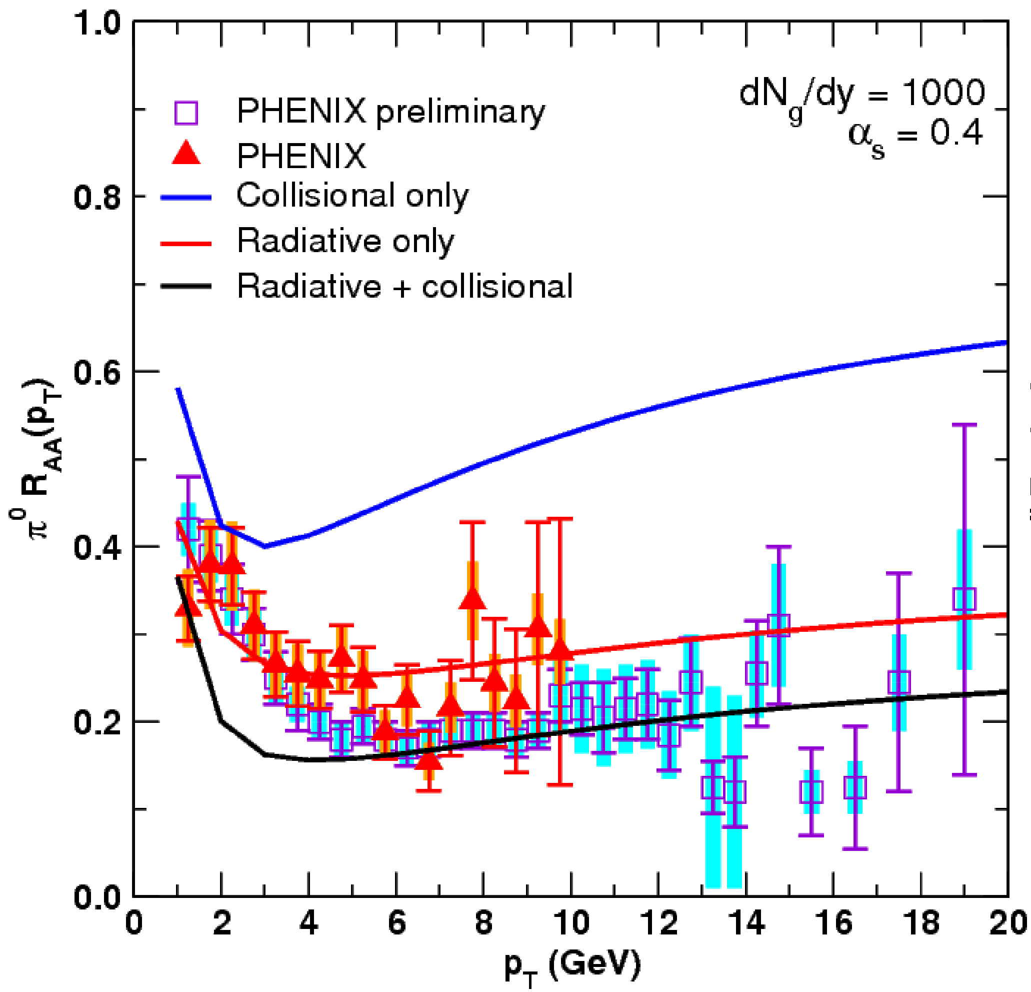
The medium

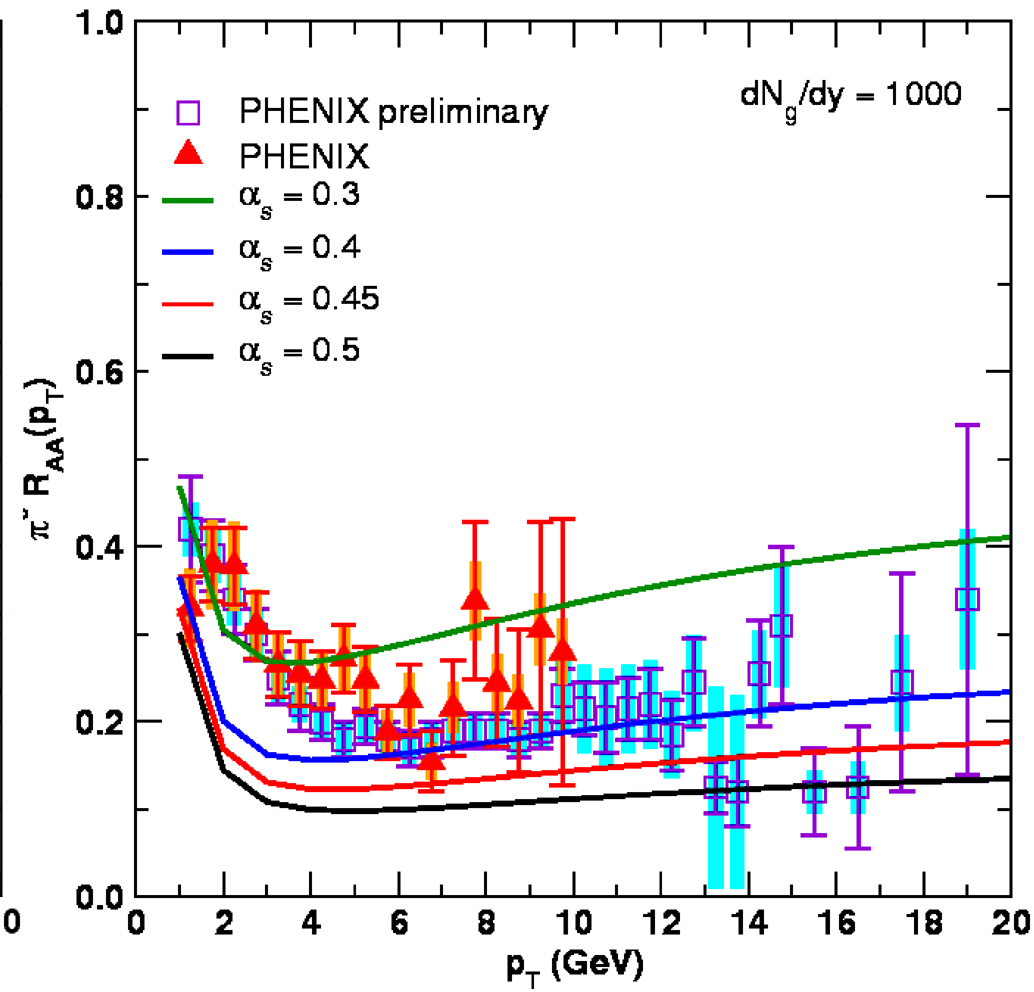
- Bulk: distributed by participant density
- Jets: distributed by binary density
- Bjorken expansion: implement by `L/2' approximation.



Results - RHIC

Pions

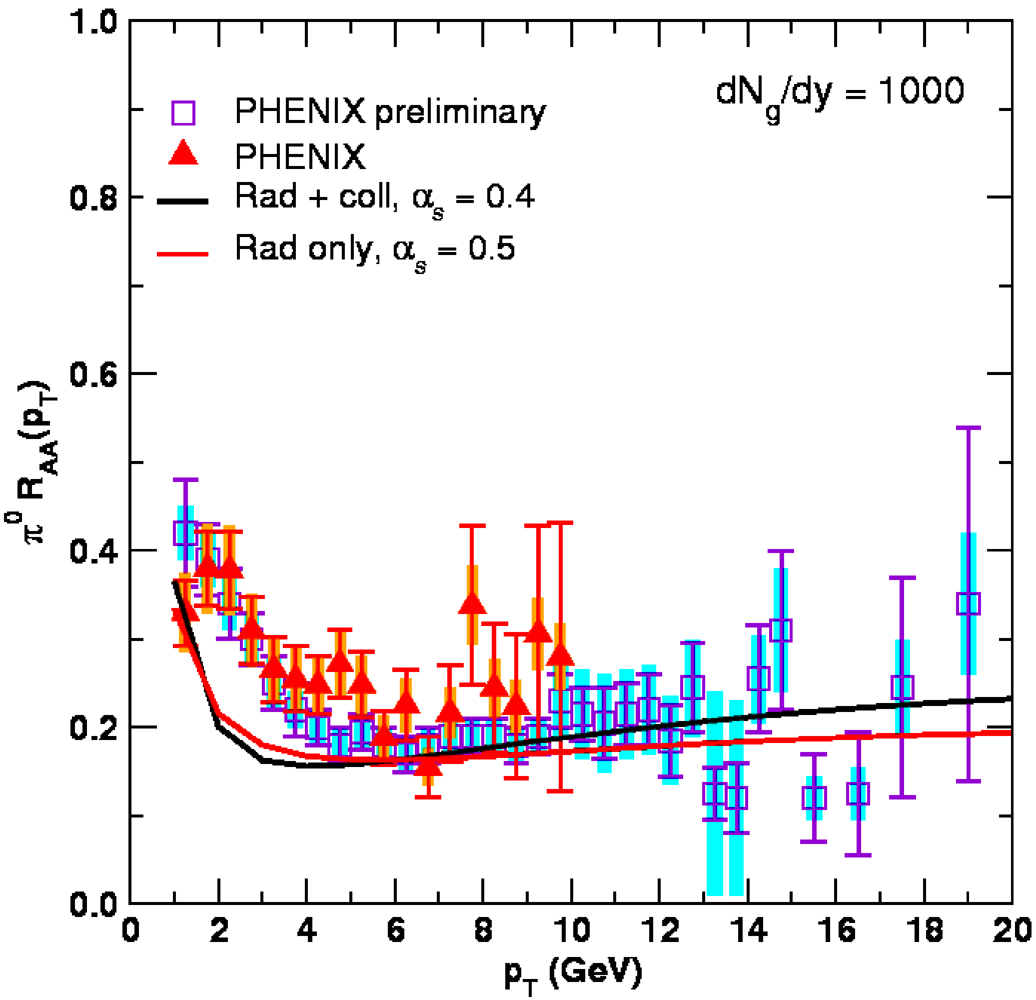




Predicting LHC

Pions

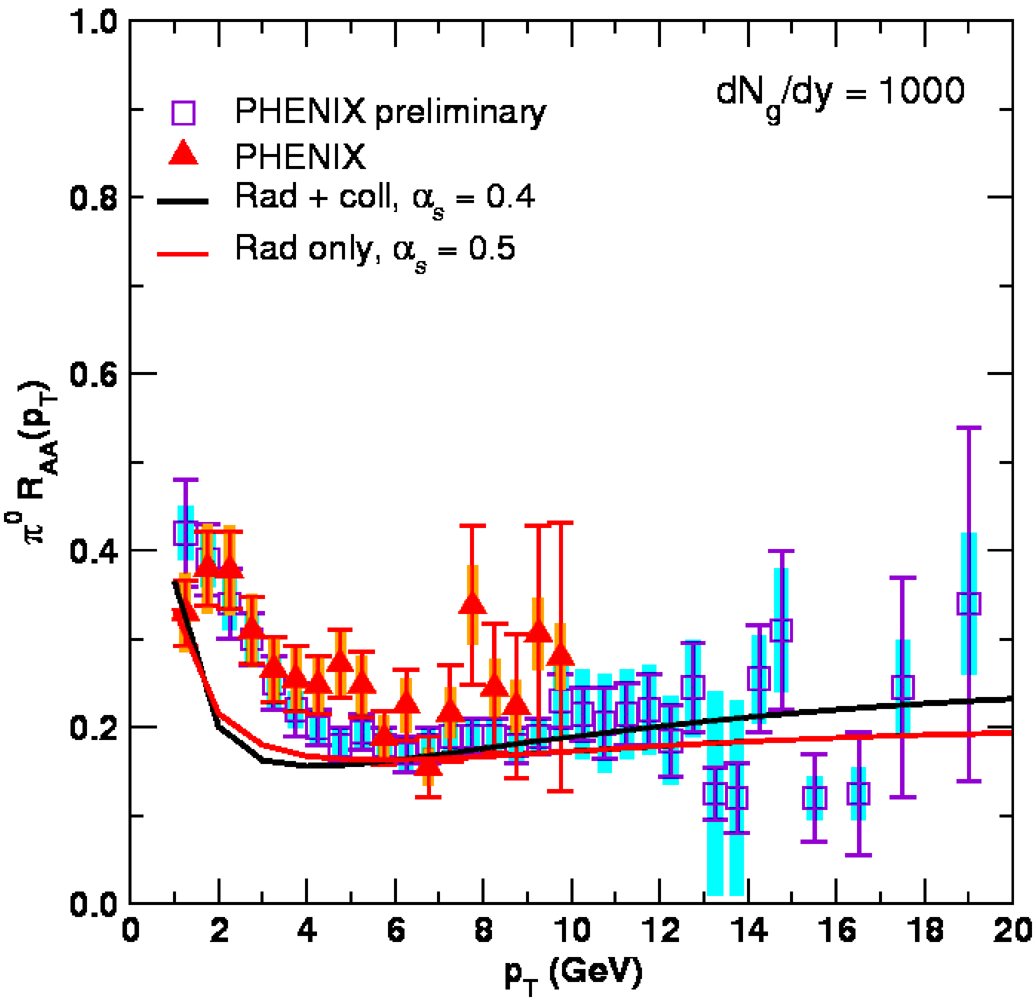
RHIC



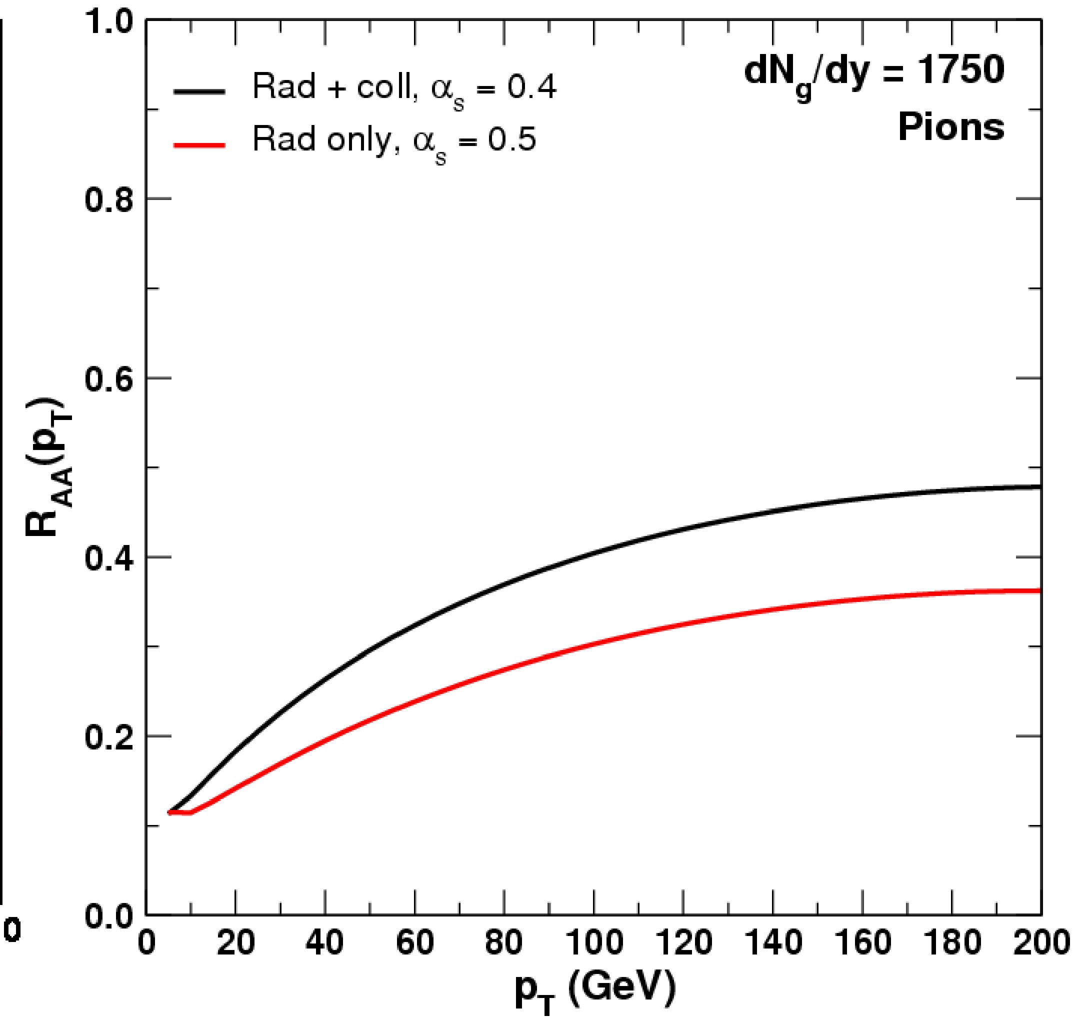
Predicting LHC

Pions

RHIC

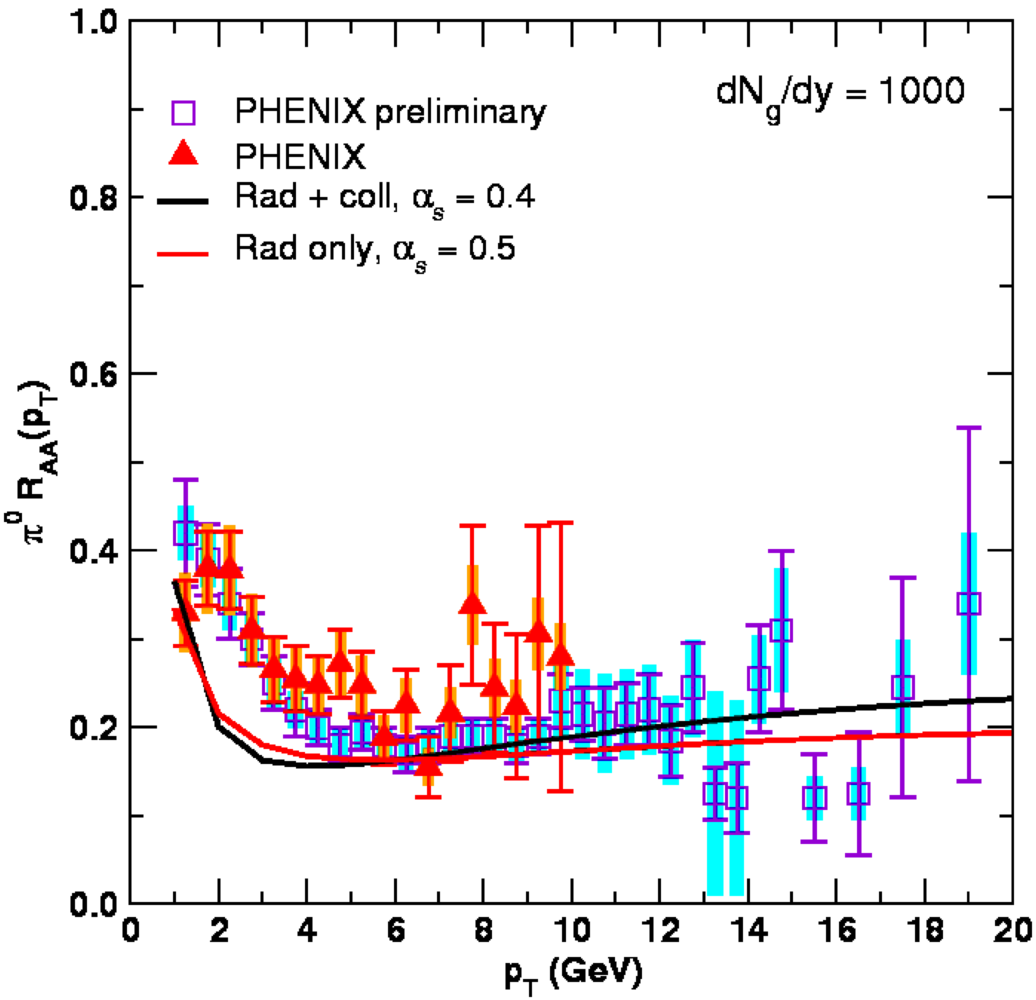


LHC

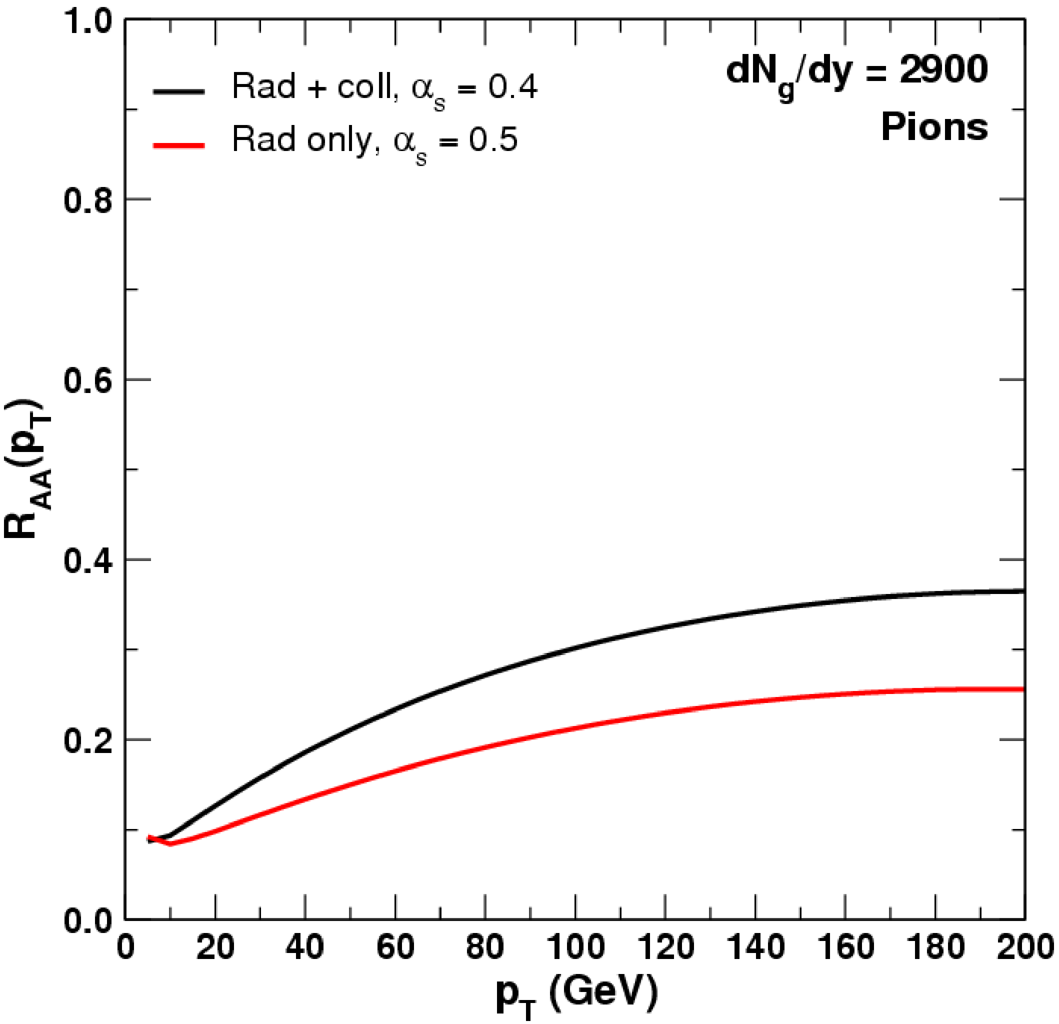


Predicting LHC
Pions

RHIC

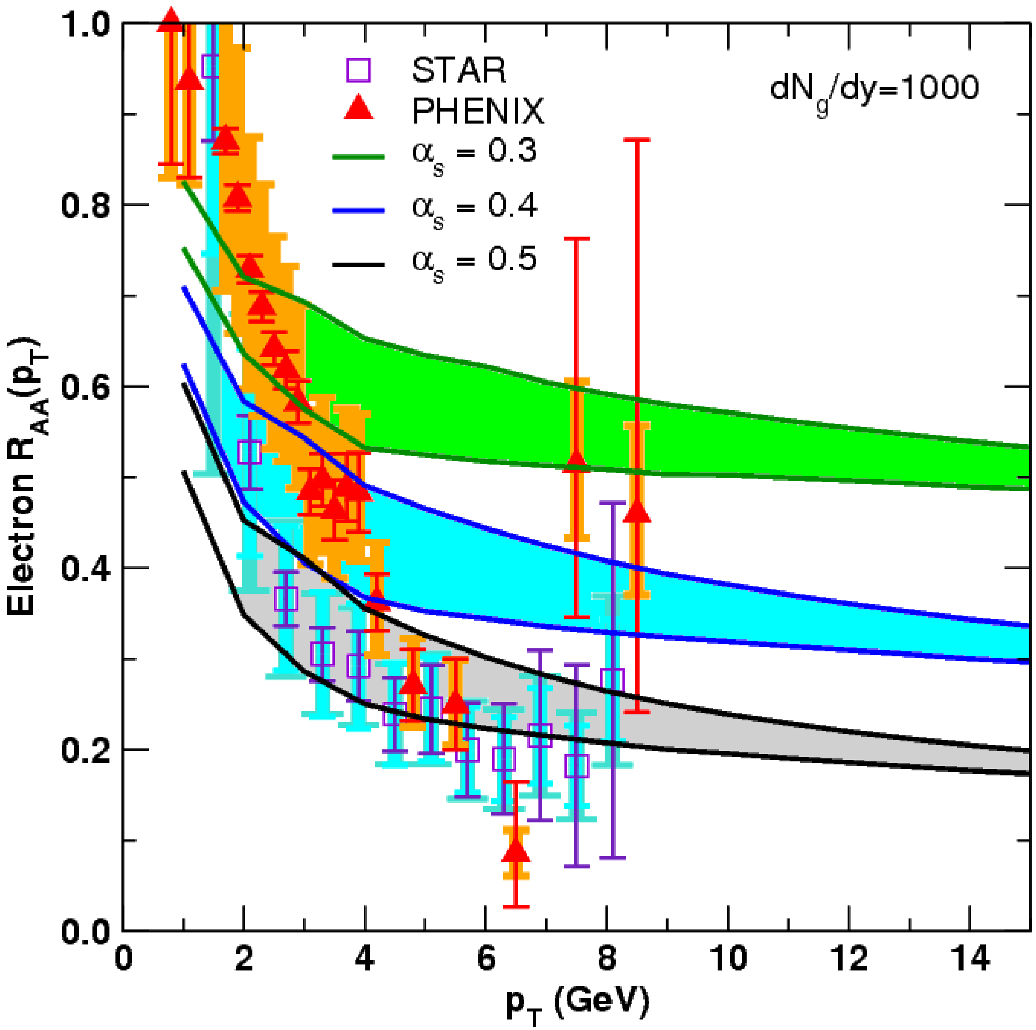


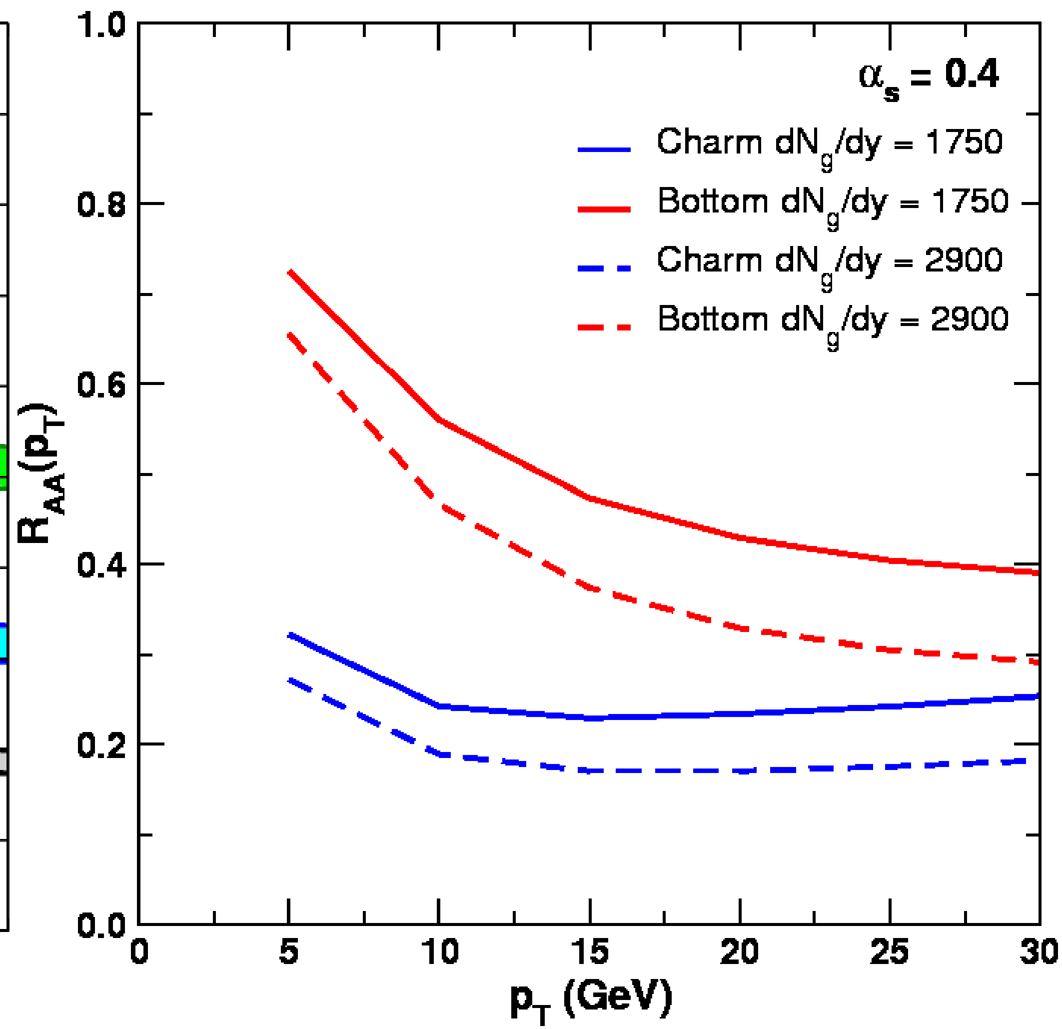
LHC



Predicting - LHC

Heavy quarks

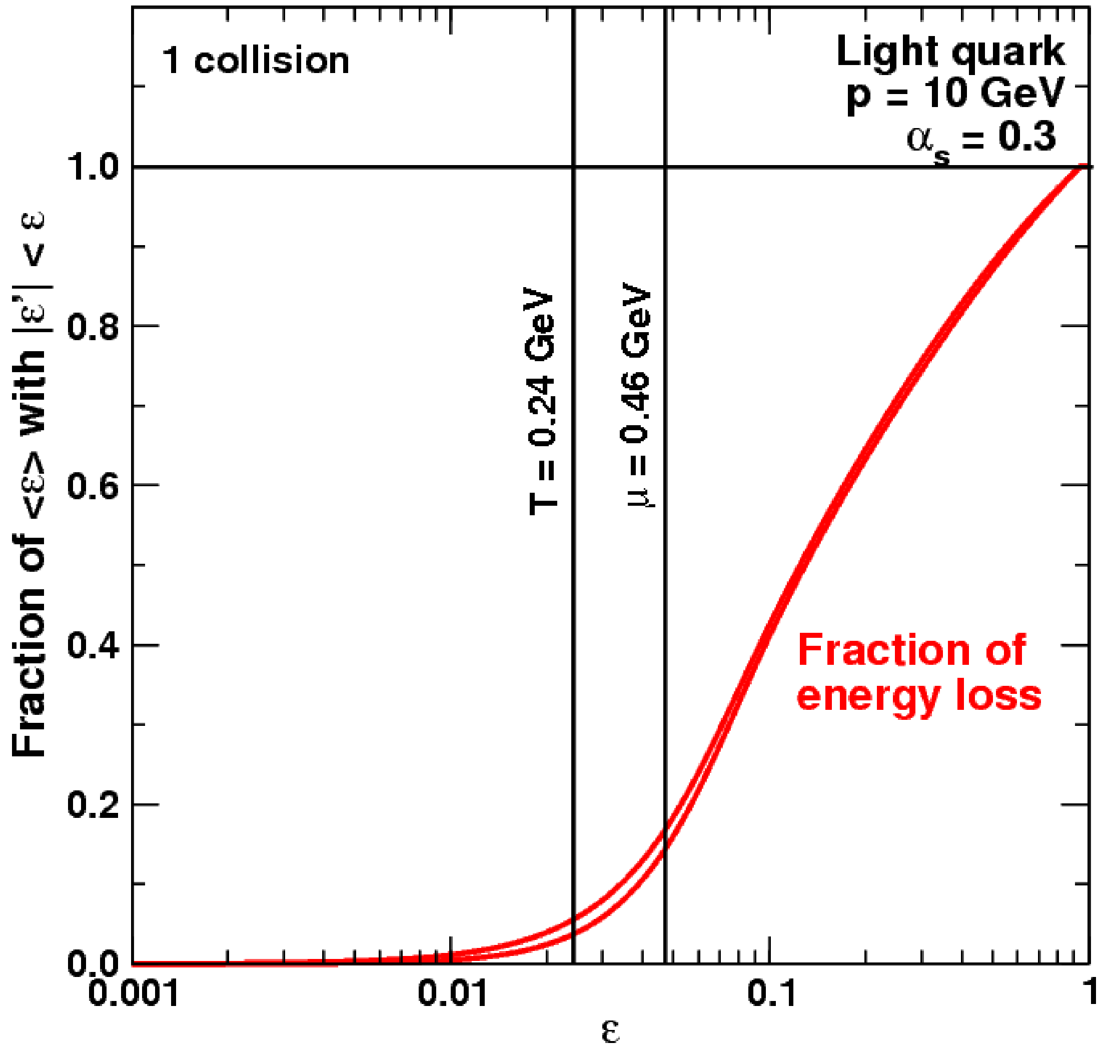


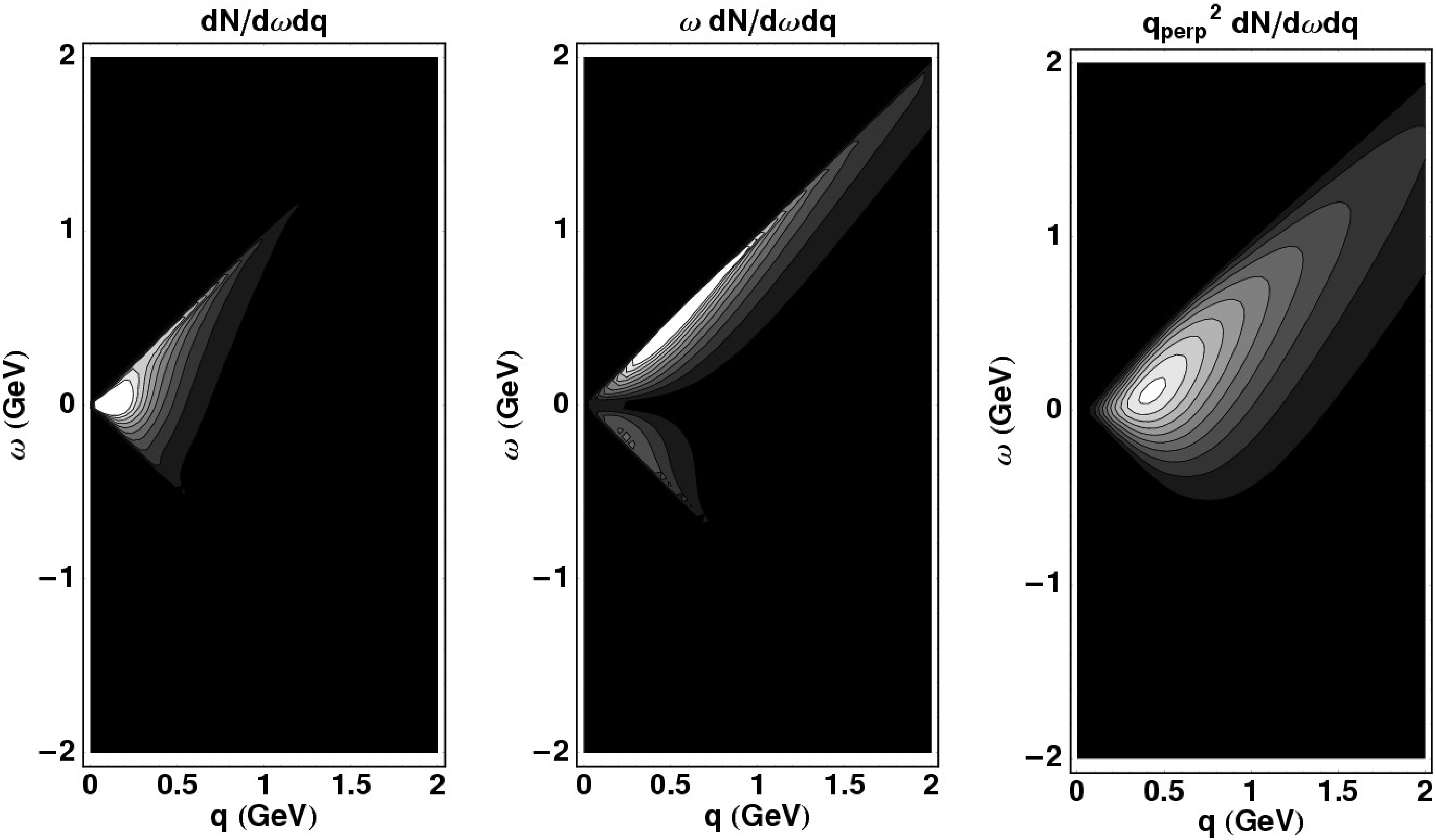


A closer look …

… at collisional energy loss and the uncertainties.

The uncertainties




dN/dωdq
ω dN/dωdq
$q_{perp}{}^2$ dN/dωdq
ω (GeV)
q (GeV)
2
1
0
−1
−2
0
0.5
1
1.5
2

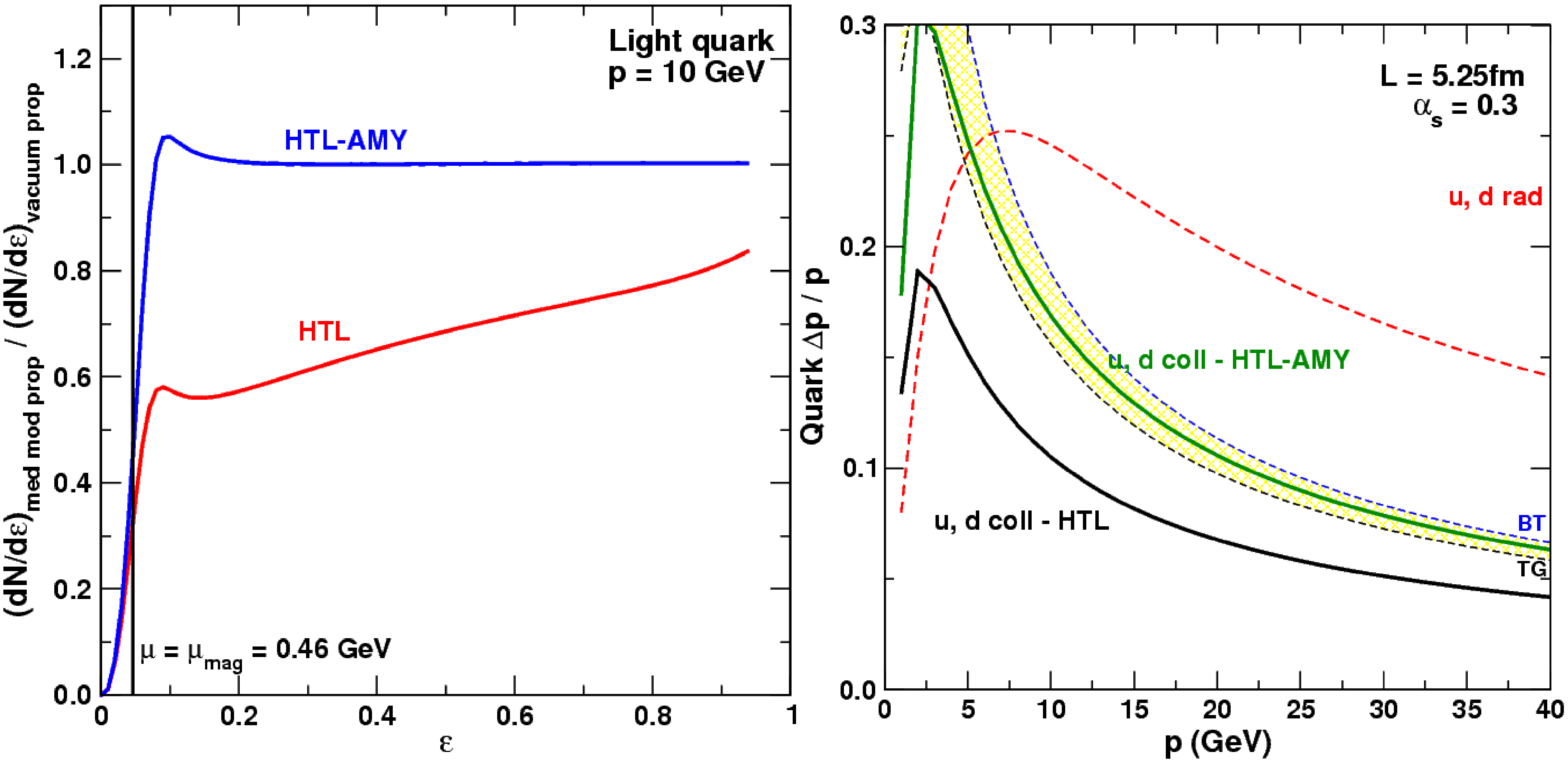
How to quantify?

Look at two schemes that are equivalent 'at leading order'

Both agree in limit $\omega, q \ll T, \mu$ and in limit $\omega, q \to \infty$ (or $\mu \to 0$)

1) Simple extrapolation of HTL to large momentum transfer.
2) Prescription found in AMY – only modify infrared divergent part of amplitude.

Result



Medium effects can persist out to high momentum exchange (as close to light cone)

Equivalent at leading order

$$\frac{\langle \Delta E \rangle_{HTL-AMY}}{\langle \Delta E \rangle_{HTL}} =$$

g = 2, pt = 10GeV: **1.6**
g = 1, pt = 10GeV: 1.3

g = 0.1, pt = 1GeV: 1.0

Conclusions

- Radiative energy loss is main contribution at LHC
  - BUT collisional energy loss affects fitting to RHIC, hence extrapolation to LHC
- Diffusion (continuum) process not applicable to length scales of interest
- Need information in region $\omega > T,\mu$ to make predictions
  - hence, 'leading order' HTL gives ~ 50% uncertainty
  - what is uncertainty in coll / rad ratio?
- The medium can affect high momentum exchange processes
  - if close to the light cone

Phase space

$$\frac{1}{2p^0}\int \frac{d^3k}{(2\pi)^3 2k^0}\frac{d^3k'}{(2\pi)^3 2k'^0}\frac{d^3p'}{(2\pi)^3 2p'^0}(2\pi)^4\delta^4(P+K-P'-K')$$

Similar treatment to:
Moore & Teaney Phys.Rev.C71:064904,2005
Djordjevic Phys.Rev.C74:064907,2006
Arnold, Moore, Yaffe JHEP 0305:051,2003

$$\begin{aligned}\delta(\omega+E-E') &= \frac{E'}{pq}\delta\left(\cos\theta_{pq}-\left(\frac{\omega}{vq}+\frac{\omega^2-q^2}{2pq}\right)\right)\\ \delta(\omega-E_k+E'_k) &= \frac{E'_k}{kq}\delta\left(\cos\theta_{kq}-\left(\frac{\omega}{v_k q}-\frac{\omega^2-q^2}{2kq}\right)\right)\end{aligned}$$

Massive jet, massless medium

$$\frac{dN}{d\omega}=\frac{1}{E^2}\frac{1}{v}\frac{1}{(2\pi)^4}\int_{|p-\sqrt{(\omega+E)^2-M^2}|}^{p+\sqrt{(\omega+E)^2-M^2}} dq\int_{\frac{1}{2}(q+\omega)}^{\infty} dk\int_0^{2\pi} d\phi\,(\ldots)$$

The Matrix Element

$$\frac{1}{2p^0}\int \frac{d^3k}{(2\pi)^3 2k^0}\frac{d^3k'}{(2\pi)^3 2k'^0}\frac{d^3p'}{(2\pi)^3 2p'^0}(2\pi)^4\delta^4(P+K-P'-K')$$

$$\begin{aligned}\delta(\omega+E-E') &= \frac{E'}{pq}\delta\left(\cos\theta_{pq}-\left(\frac{\omega}{vq}+\frac{\omega^2-q^2}{2pq}\right)\right)\\ \delta(\omega-E_k+E_k') &= \frac{E_k'}{kq}\delta\left(\cos\theta_{kq}-\left(\frac{\omega}{v_k q}-\frac{\omega^2-q^2}{2kq}\right)\right)\end{aligned}$$

$$\frac{1}{v}\frac{1}{(2\pi)^4}\int_{|p-\sqrt{(\omega+E)^2-M^2}|}^{p+\sqrt{(\omega+E)^2-M^2}} dq\int_{\frac{1}{2}(q+\omega)}^{\infty} dk\int_0^{2\pi} d\phi\,(\ldots)$$

$$\langle |M|^2\rangle = 4g^4 k_{CF}\left(p^\mu p^{\mu'}+p^{\mu'}p'^{\mu}+(M^2-p.p')g^{\mu\mu'}\right)D_{\mu\nu}(q)D^*_{\mu'\nu'}(q)\left(k^\nu k'^{\nu'}+k^{\nu'}k'^{\nu}+(m^2-k.k')g^{\nu\nu'}\right)$$

$$D_{\mu\nu}(q) = Q_{\mu\nu}(q)\Delta_L(q)+P_{\mu\nu}\Delta_T(q)$$

$$Q_{00}=1 \quad , \quad \Delta_L(Q)=\frac{1}{q^2-\Pi_l}$$

$$P_{ij}=-(g_{ij}+\hat{q}_i\hat{q}_j) \quad , \quad \Delta_T(Q)=\frac{1}{\omega^2-q^2-\Pi_t}$$

$$\langle |M|^2\rangle = 16g^4 k_{CF}E^2E_k^2\left[C_{LL}|\Delta_L(q)|^2+2C_{LT}Re(\Delta_L(q)\Delta_T^*(q))+C_{TT}|\Delta_T(q)|^2\right]$$

$$\begin{aligned}C_{LL} &= \left((1+\frac{\omega}{2E})^2-\frac{q^2}{4E^2}\right)\left((1-\frac{\omega}{2E_k})^2-\frac{q^2}{4E_k^2}\right)\\ C_{LT} &= 0\\ C_{TT} &= \frac{1}{2}\left(v^2-\frac{\omega^2}{q^2}\left(1+\frac{\omega}{2E}\right)^2+\frac{1}{E}\left(\omega+\frac{q^2}{4E}\right)\right)\left(v_k^2-\frac{\omega^2}{q^2}\left(1-\frac{\omega}{2E_k}\right)^2-\frac{1}{E_k}\left(\omega-\frac{q^2}{4E_k}\right)\right)\end{aligned}$$

The Matrix Element (cont.)

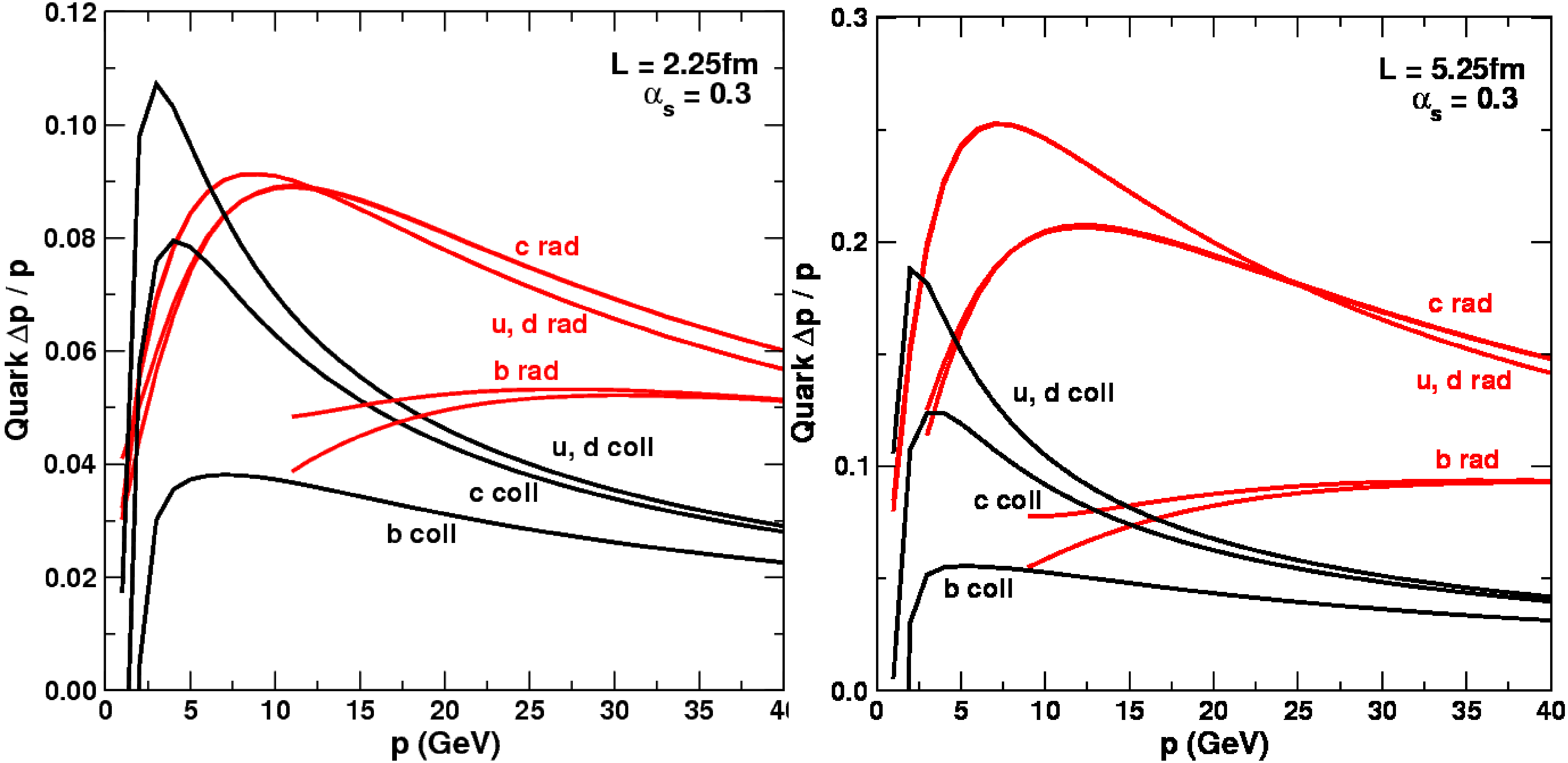
$$\Delta_L(q) = \left[q^2 + \mu^2(1 + \frac{\omega}{2q}\ln(\frac{\omega - q}{\omega + q}))\right]^{-1}$$

$$\Delta_T(q) = \left[\omega^2 - q^2 - \frac{\mu^2}{2} - \frac{(\omega^2 - q^2)\mu^2}{2q^2}(1 + \frac{\omega}{2q}\ln(\frac{\omega - q}{\omega + q}))\right]^{-1}$$

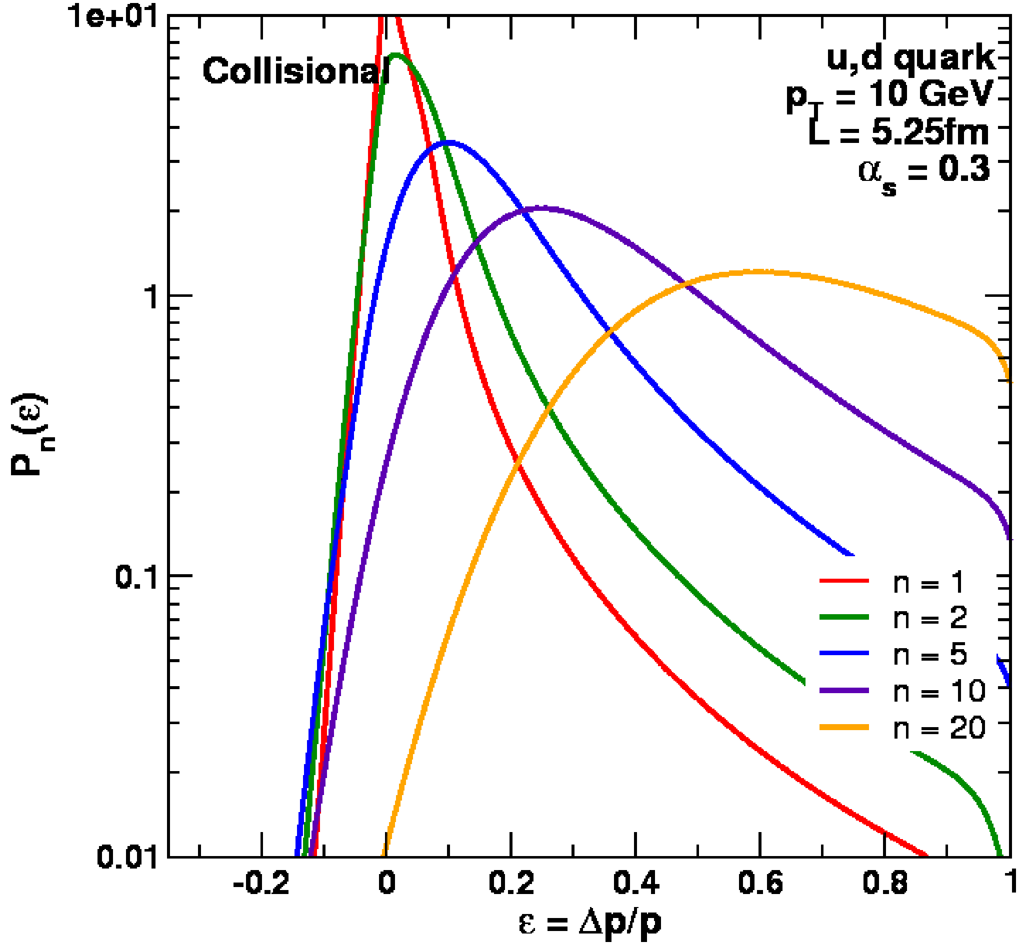
After $\int d\phi/2\pi$ integral, where ϕ is the angle between the (pq) and (kq) planes.

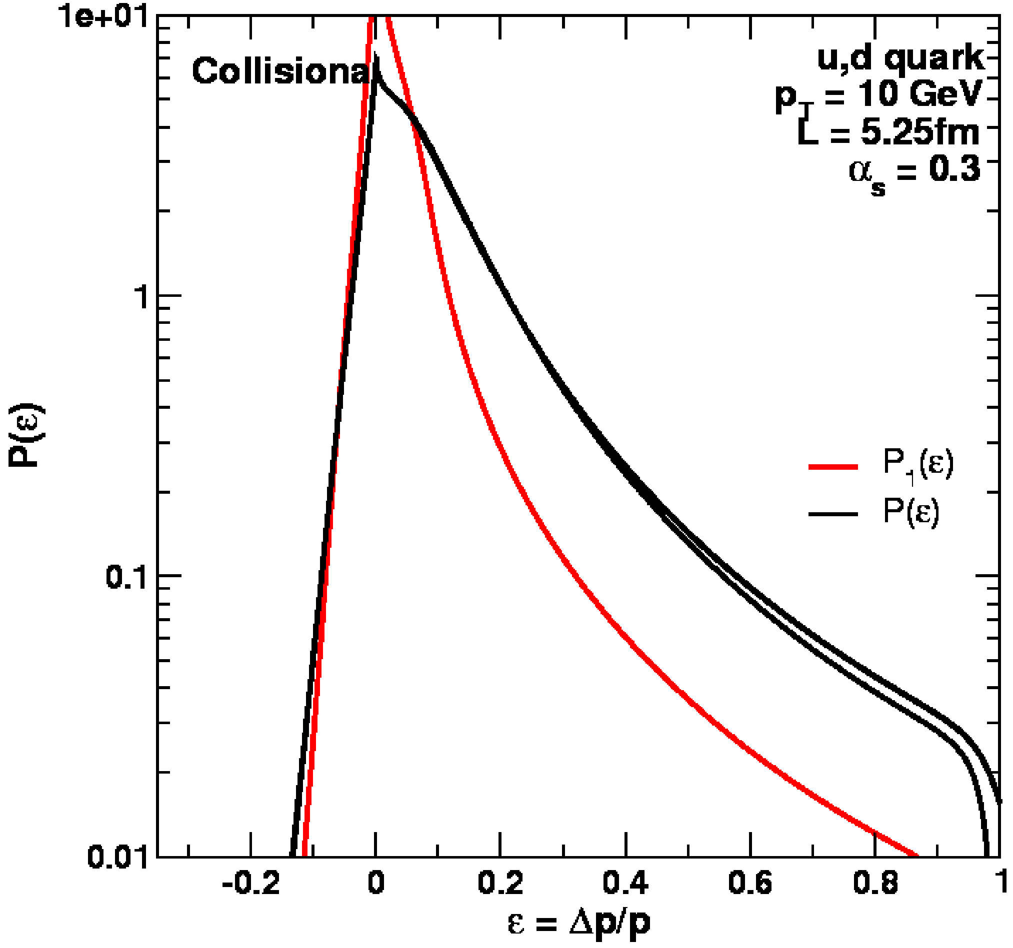
$$\begin{aligned}
C_{LL} &= \left((1+\frac{\omega}{2E})^2 - \frac{q^2}{4E^2}\right)\left((1-\frac{\omega}{2E_k})^2 - \frac{q^2}{4E_k^2}\right) \\
C_{LT} &= 0 \\
C_{TT} &= \frac{1}{2}\left(v^2 - \frac{\omega^2}{q^2}\left(1+\frac{\omega}{2E}\right)^2 + \frac{1}{E}\left(\omega + \frac{q^2}{4E}\right)\right)\left(v_k^2 - \frac{\omega^2}{q^2}\left(1-\frac{\omega}{2E_k}\right)^2 - \frac{1}{E_k}\left(\omega - \frac{q^2}{4E_k}\right)\right)
\end{aligned}$$

Average energy loss



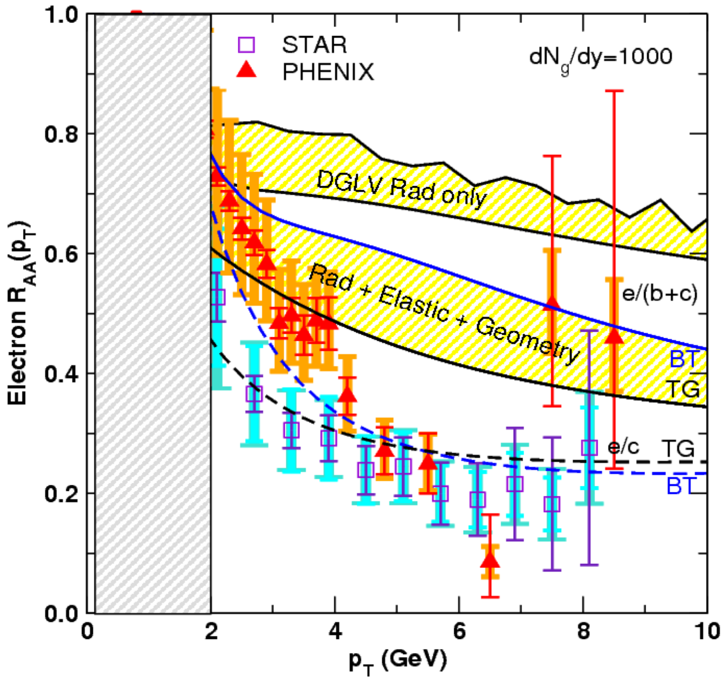
Multiple Collisions



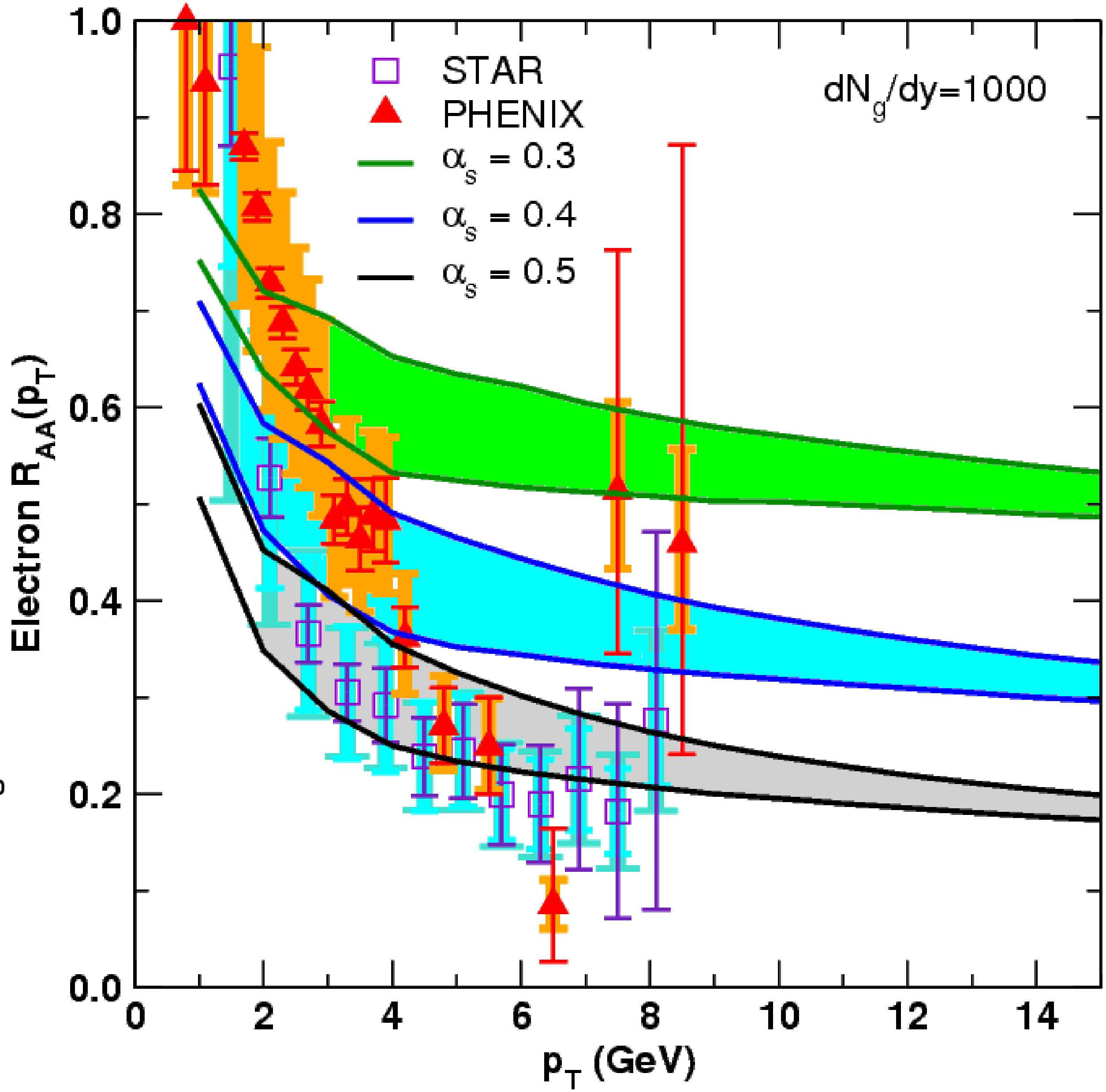


NOT continuum limit diffusion process

Results – RHIC - Electrons



WHDG α = 0.3



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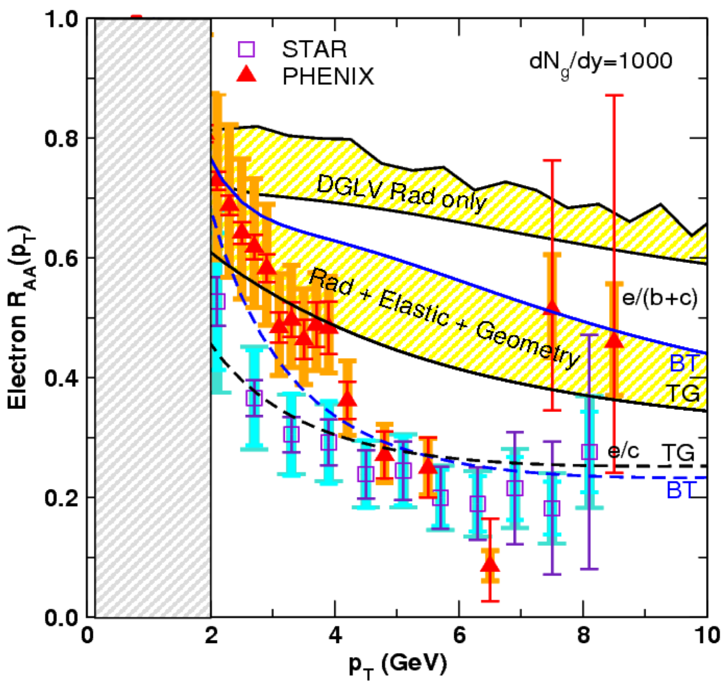

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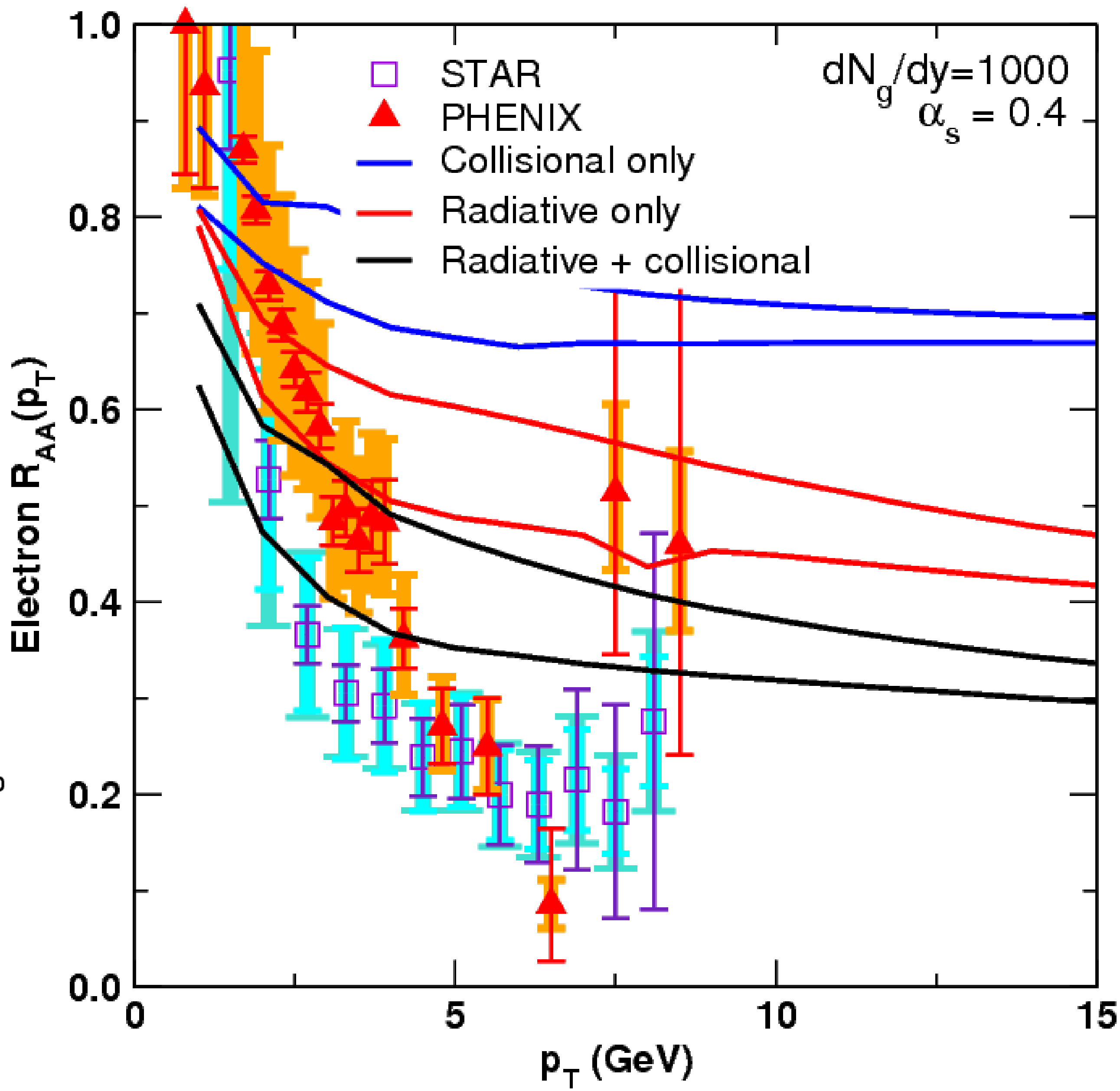
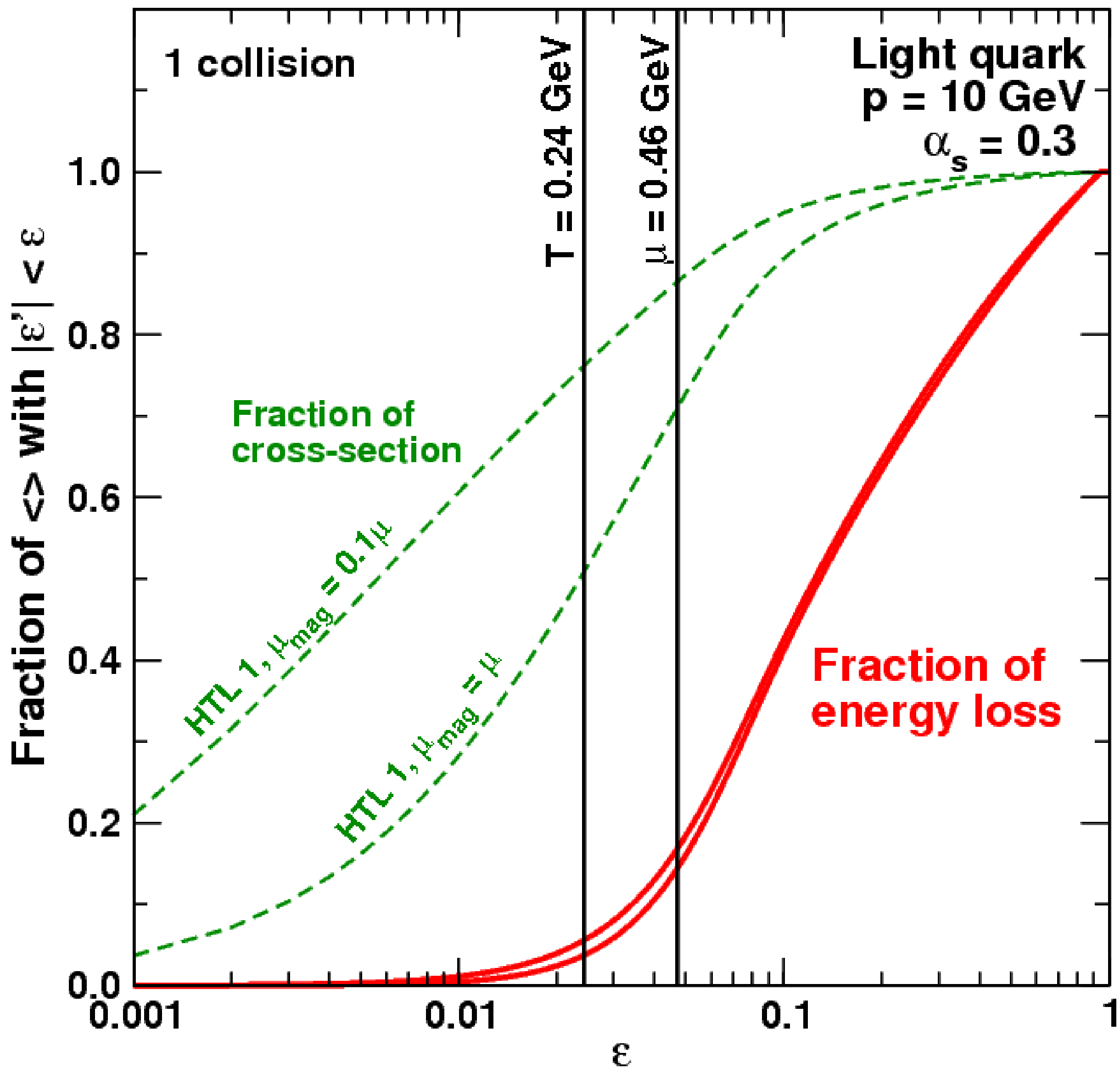


ω << T, μ assumption / approximation is NOT ok to calculate av en loss

Must take into account medium recoil.



HTL extrapolation

$$\langle |M|^2 \rangle = 16 g^4 k_{CF} E^2 E_k^2 \left[C_{LL} |\Delta_L(q)|^2 + 2 C_{LT} Re(\Delta_L(q) \Delta_T^*(q)) + C_{TT} |\Delta_T(q)|^2 \right]$$

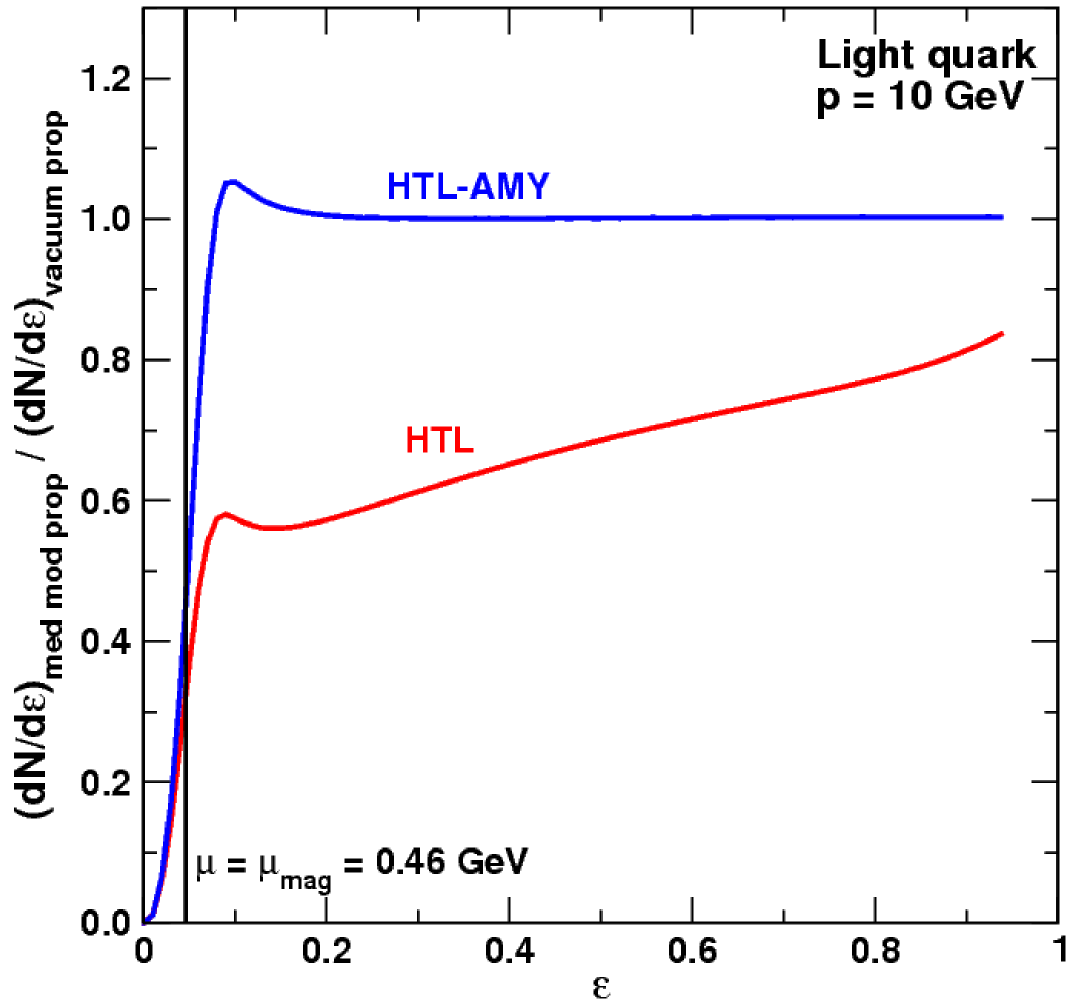
$$\begin{aligned}
\boldsymbol{C_{LL}} &= \left((1 + \frac{\omega}{2E})^2 - \frac{q^2}{4E^2} \right) \left((1 - \frac{\omega}{2E_k})^2 - \frac{q^2}{4E_k^2} \right) \\
\boldsymbol{C_{LT}} &= 0 \\
\boldsymbol{C_{TT}} &= \tfrac{1}{2} \left(v^2 - \frac{\omega^2}{q^2} \left(1 + \frac{\omega}{2E} \right)^2 + \frac{1}{E} \left(\omega + \frac{q^2}{4E} \right) \right) \left(v_k^2 - \frac{\omega^2}{q^2} \left(1 - \frac{\omega}{2E_k} \right)^2 - \frac{1}{E_k} \left(\omega - \frac{q^2}{4E_k} \right) \right)
\end{aligned}$$

HTL-AMY extrapolation

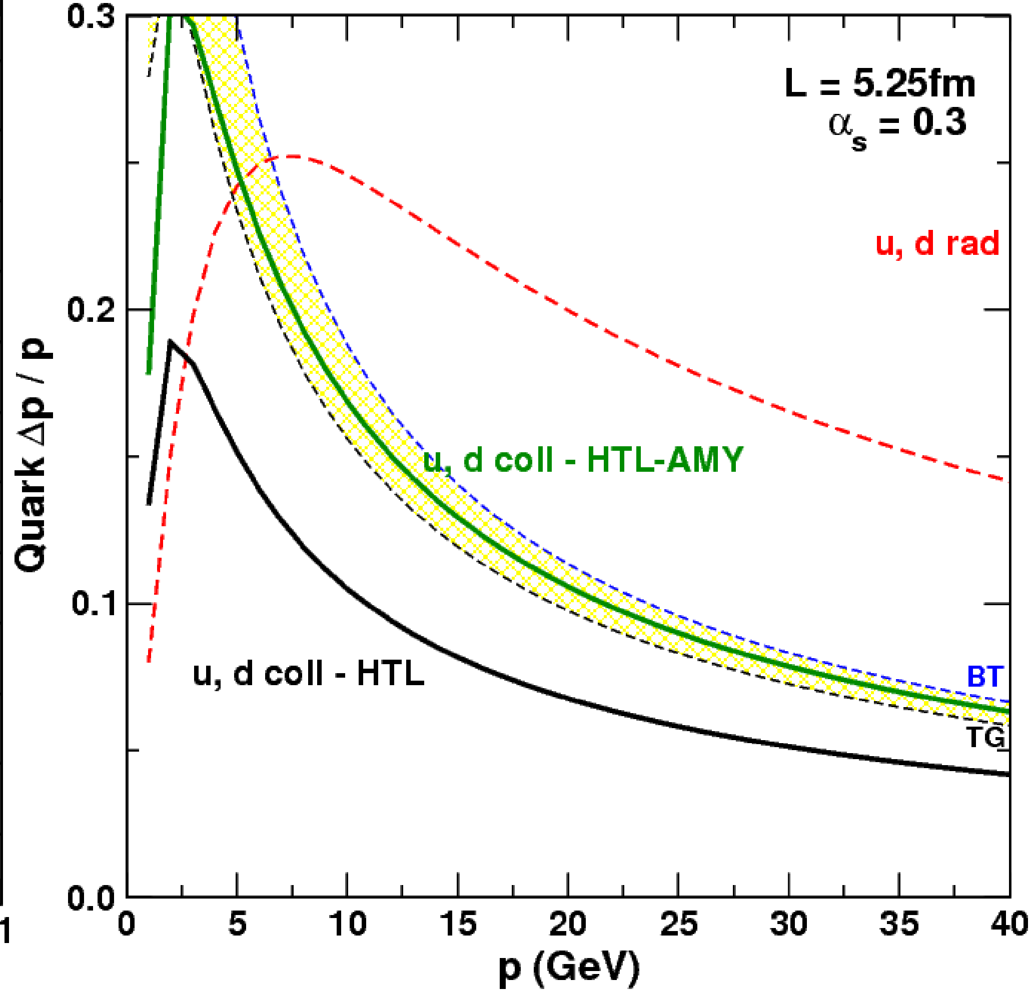
$$\langle |M|^2 \rangle = g^4 k_{CF} + 16 g^4 k_{CF} E^2 E_k^2 \left[C_{LL} |\Delta_L(q)|^2 + 2 C_{LT} Re(\Delta_L(q) \Delta_T^*(q)) + C_{TT} |\Delta_T(q)|^2 \right]$$

$$\begin{aligned}
C_{LL} &= \left(1 + \frac{\omega}{2E} \right)^2 \left(1 - \frac{\omega}{2E_k} \right)^2 \\
C_{LT} &= 0 \\
C_{TT} &= \tfrac{1}{2} \left(v^2 - \left(\frac{\omega}{q} + \frac{\omega^2 - q^2}{2Eq} \right)^2 \right) \left(v_k^2 - \left(\frac{\omega}{q} - \frac{\omega^2 - q^2}{2E_k q} \right)^2 \right)
\end{aligned}$$

Result



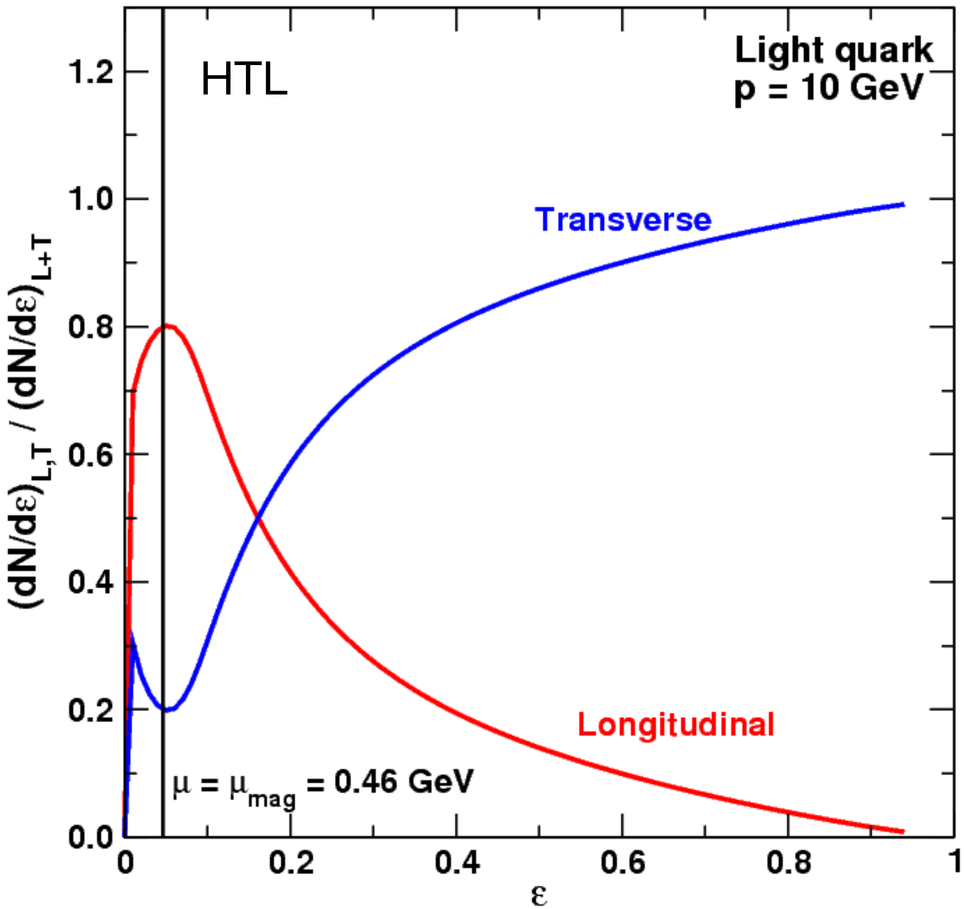
Light quark
p = 10 GeV
HTL-AMY
HTL
$\mu = \mu_{mag} = 0.46$ GeV
$(dN/d\varepsilon)_{med\ mod\ prop} / (dN/d\varepsilon)_{vacuum\ prop}$
ε

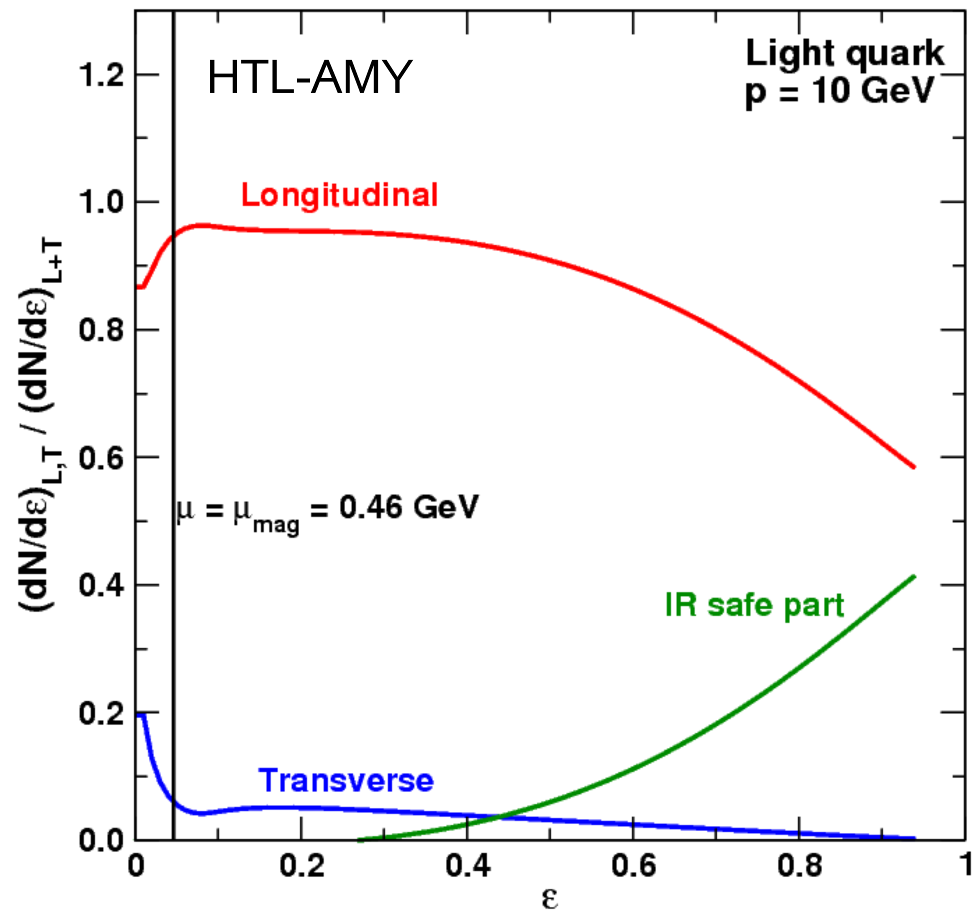


L = 5.25fm
$\alpha_s = 0.3$
u, d rad
u, d coll - HTL-AMY
u, d coll - HTL
BT
TG
Quark Δp / p
p (GeV)

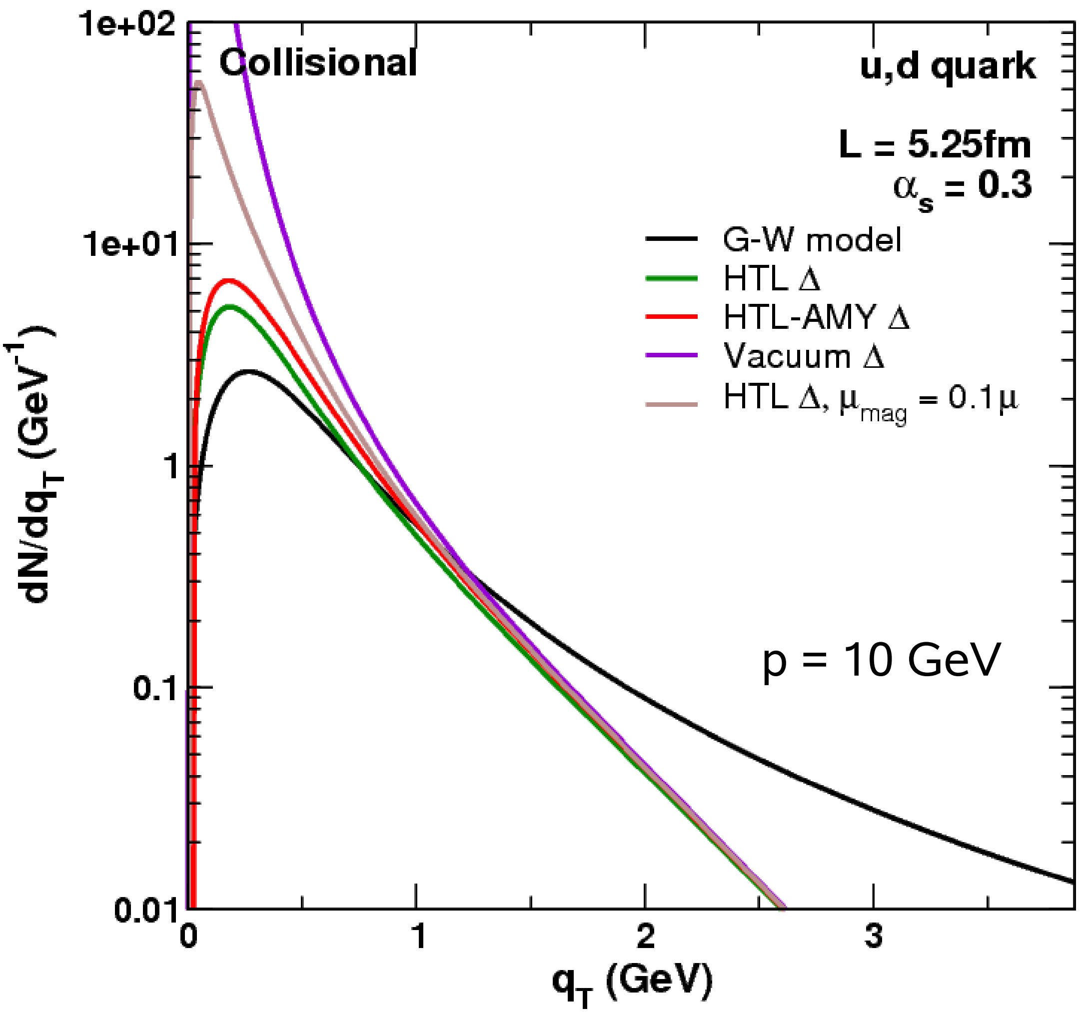
Why are HTL and HTL-AMY so different?

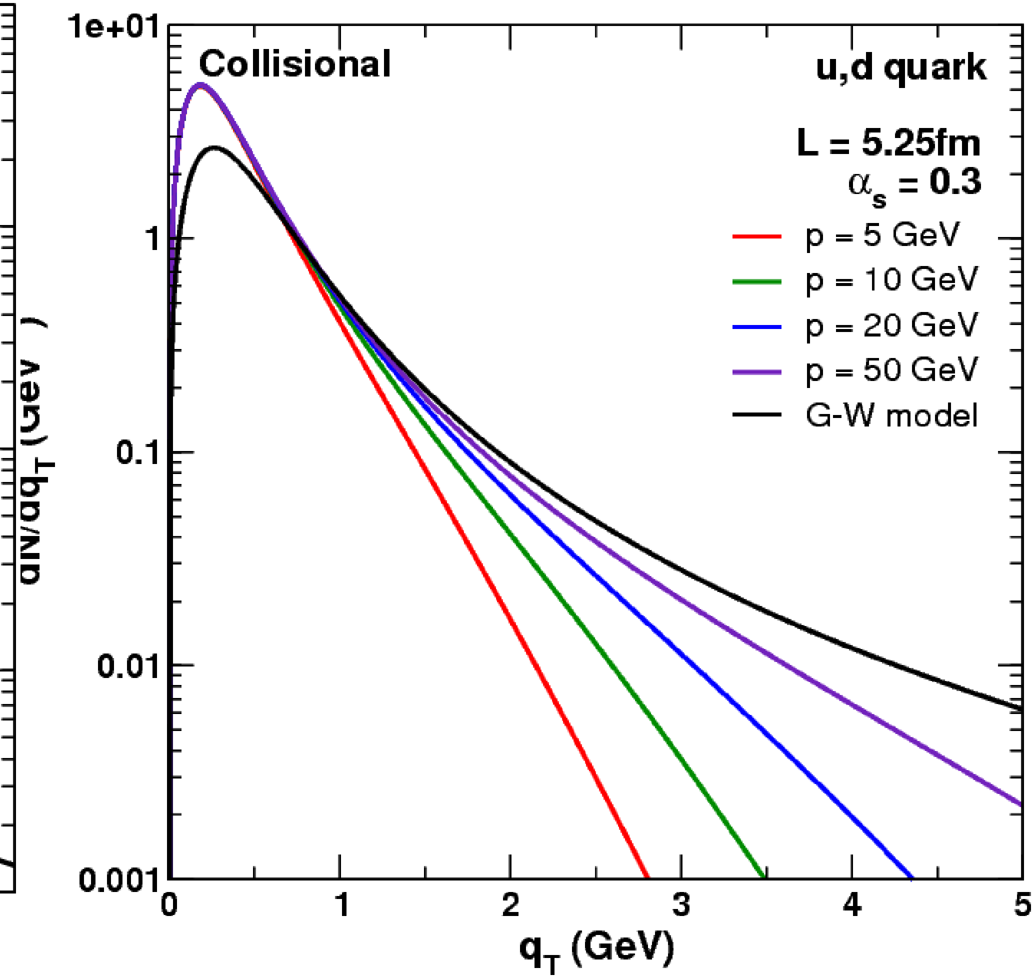
Redistribution of longitudinal and transverse components.
Longitudinal and transverse components are screened by the medium in different ways.





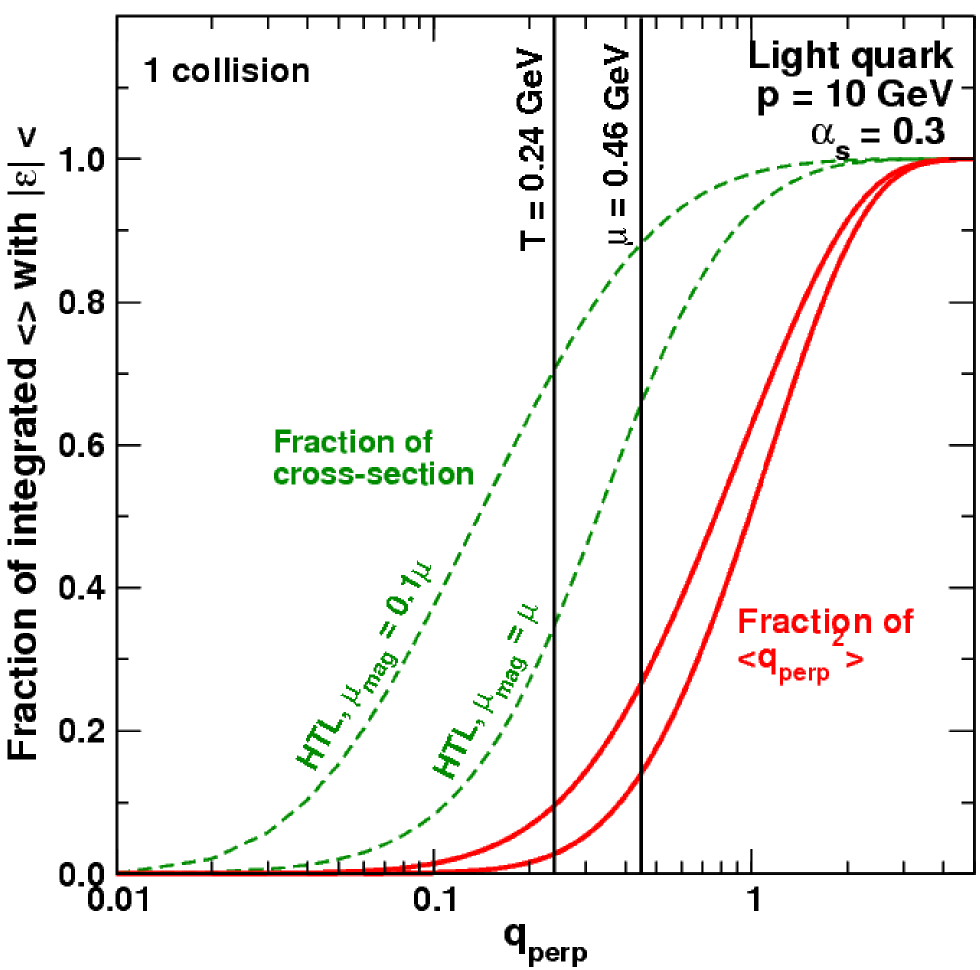
What about q_{perp} distributions?

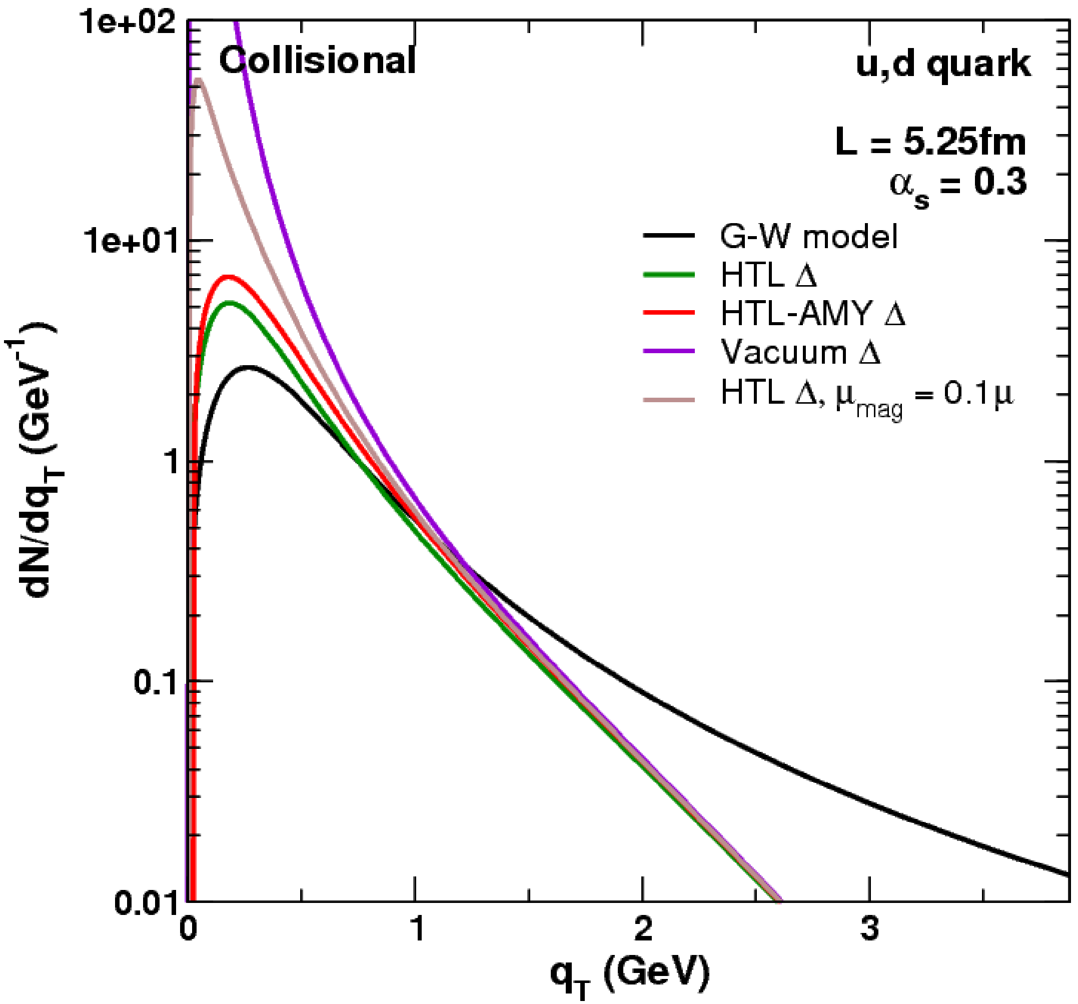




p=10GeV: $\langle q_{perp}^2 \rangle \approx 0.25$ GeV^2/fm for T = 0.24GeV

What about q_{perp} distributions?





The rare, hard collisions contribute most to $\langle q_{perp}^2 \rangle$

If radiation is driven by $\langle q_{perp}{}^2 \rangle$, then we are not in the regime where:

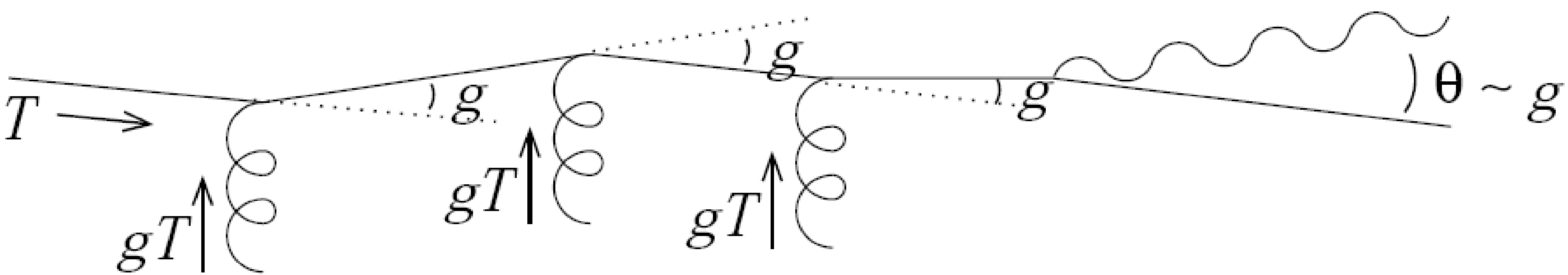


Diagram from Arnold, Moore and Yaffe: JHEP 0206:030,2002